Syllabus structure

Core Courses

The following courses are compulsory for BSc Mathematics programme.

Sl. No	Code	Name of the course	Semester	No of contact hours/Week	Credits	Max. Marks			Exam dur.(Hrs)
						Internal	External	Total	
1	MTS1B01	Basic Logic and Number Theory	1	4	4	20	80	100	2.5
2	MTS2B02	Calculus of Single variable-1	2	4	4	20	80	100	2.5
3	MTS3B03	Calculus of Single variable-2	3	5	4	20	80	100	2.5
4	MTS4B04	Linear Algebra	4	5	4	20	80	100	2.5
5	MTS5B05	Abstract Algebra	5	5	4	20	80	100	2.5
6	MTS5B06	Basic Analysis	5	5	4	20	80	100	2.5
7	MTS5B07	Numerical Analysis	5	4	3	15	60	75	2
8	MTS5B08	Linear Programming	5	3	3	15	60	75	2
9	MTS5B09	Introduction to Geometry and Theory of Equations	5	3	3	15	60	75	2
		Project	5	2					
10		Open Course (Offered by Other Departments)	5	3	3	15	60	75	2
11	MTS6B10	Real Analysis	6	5	5	20	80	100	2.5
12	MTS6B11	Complex Analysis	6	5	5	20	80	100	2.5
13	MTS6B12	Calculus of Multi variable	6	5	4	20	80	100	2.5
14	MTS6B13	Differential Equations	6	5	4	20	80	100	2.5
15	MTS6B14	Elective	6	3	2	15	60	75	2
16	MTS6P15(PR)	Project Viva	6	2	2	15	60	75	
				68	58			1450	

FIRST SEMESTER

MTS1 B01 BASIC LOGIC & NUMBER THEORY

4 hours/week	4 Credits	100 Marks Int:20+Ext:80]

Aims, Objectives and Outcomes	

Logic, the study of principles of techniques and reasoning, is fundamental to every branch of learning. Besides, being the basis of all mathematical reasoning, it is required in the field of computer science for developing programming languages and also to check the correctness of the programmes. Electronic engineers apply logic in the design of computer chips. The first module discusses the fundamentals of logic, its symbols and rules. This enables one to think systematically, to express ideas in precise and concise mathematical terms and also to make valid arguments. How to use logic to arrive at the correct conclusion in the midst of confusing and contradictory statements is also illustrated.

The classical number theory is introduced and some of the very fundamental results are discussed in other modules. It is hoped that the method of writing a formal proof, using proof methods discussed in the first module, is best taught in a concrete setting, rather than as an abstract exercise in logic. Number theory, unlike other topics such as geometry and analysis, doesn't suffer from too much abstraction and the consequent difficulty in conceptual understanding. Hence, it is an ideal topic for a beginner to illustrate how mathematicians do their normal business. By the end of the course, the students will be able to enjoy and master several techniques of problem solving such as recursion, induction etc., the importance of pattern recognition in mathematics, the art of conjecturing and a few applications of number theory. Enthusiastic students will have acquired knowledge to read and enjoy on their own a few applications of number theory in the field of art, geometry and coding theory. Successful completion of the course enables students to

- Prove results involving divisibility, greatest common divisor, least common multiple and a few applications.
- Understand the theory and method of solutions of LDE.
- Solve linear congruent equations.
- Learn three classical theorems *viz.* Wilson's theorem, Fermat's little theorem and Euler's theorem and a few important consequences.

Syllabus

Text (1)	Discrete Mathematics with Applications : Thomas Koshy, *Elsever Academic Press(2004) ISBN:0-12-421180-1*
Text:(2)	Elementary Number Theory with Applications (2/e) :Thomas Koshy, *Elsever Academic Press(2007) ISBN:978-0-12-372487-8*

Module- I Text (1) (15 hrs)

1.1: Propositions- definition, Boolean (logic) variables, Truth Value, Conjunction, Boolean expression, Disjunction (inclusive and exclusive), Negation, Implication, Converse, Inverse and Contra positive, Biconditional statement, Order of Precedence, Tautology Contradiction and Contingency [‘Switching Networks’ omitted]

1.2 : Logical equivalences- laws of logic [‘Equivalent Switching Networks’ ‘Fuzzy logic’ & ‘Fuzzy decisions’omitted]

1.3 : Quantifiers- universal & existential, predicate logic

1.4 : Arguments- valid and invalid arguments, inference rules

1.5: Proof Methods – vacuous proof, trivial proof, direct proof, indirect proof-contrapositive & contradiction, proof by cases, Existence proof- constructive & non constructive, counter example

Module- II Text (2) (12 hrs)

1.3 : Mathematical induction- well ordering principle, simple applications, weak version of principle of mathematical induction, illustrations, strong version of induction (second principle of MI), illustration

1.4 : Recursion- recursive definition of a function, illustrations.

2.1: The division algorithm – statement and proof, div & mod operator, card dealing, The two queens puzzle (simple applications), pigeonhole principle and division algorithm, divisibility relation, illustration, divisibility properties, union intersection and complement-inclusion-exclusion principle & applications, even and odd integers.

2.5: Prime and Composite Numbers- definitions, infinitude of primes, [‘algorithm 2.4‘ omitted] The sieve of Eratosthenes, a number theoretic function, prime number theorem (statement only), distribution of primes (upto and including Example 2.25) . [rest of the section omitted]

Module- III	Text (2)	(17 hrs)

3.1 : Greatest Common Divisor- gcd, symbolic definition, relatively prime integers, Duncan's identity, Polya's theorem, infinitude of primes, properties of gcd, linear combination, gcd as linear combination, an alternate definition of gcd, gcd of n positive integers, a linear combination of n positive integers, pairwise relatively prime integers, alternate proof for infinitude of prime.

3.2: The Euclidean Algorithm- The Euclidean algorithm [algorithm 3.1 omitted], A jigsaw puzzle, Lame's theorem (statement only; proof omitted)

3.3: The Fundamental Theorem of Arithmetic- Euclid's lemma on division of product by a prime, fundamental theorem of arithmetic, Canonical Decomposition, number of trailing zeros, highest power of a prime dividing!, [only statement of Theorem3.14 required; proof omitted] Distribution of Primes Revisited, Dirichlet's Theorem (statement only)

3.4 : Least Common Multiple- definition, canonical decomposition to find lcm, relationship between gcd and lcm, relatively prime numbers and their lcm

3.5: Linear Diophantine Equations – LDE in two variables, conditions to have a solution, Aryabhatta's method, number of solutions, general solution, Mahavira's puzzle, hundred fowls puzzle, Monkey and Coconuts Puzzle, ['Euler's method for solving LDE's ' omitted] Fibonacci numbers and LDE, LDE in more number of variables and their solutions- Theorem 3.20

Module- IV	Text (2)	(20 hrs)

4.1:Congruences - congruence modulo m, properties of congruence, characterization of congruence, least residue, ['Friday-the-Thirteenth' omitted], congruence classes, A Complete Set of Residues Modulo m, properties of congruence, use of congruence to find the remainder on division, ['Modular Exponentiation' method omitted], Towers of Powers Modulo m, further properties of congruence and their application to find remainder ['Monkey and Cocunut Puzzle revisited'(example 4.17) omitted] congruences of two numbers with different moduli

4.2: Linear Congruence- solvability, uniqueness of solution, incongruent solutions, Modular Inverses, applications

5.1: Divisibility Tests-Divisibility Test for 10, Divisibility Test for 5, Divisibility Test for 2^i, Divisibility Tests for 3 and 9, Divisibility Test for 11 [rest of the section from Theorem 5.1 onwards omitted]

7.1: Wilson's Theorem- self invertible modulo prime, Wilson's theorem and its converse ['Factorial,Multifactorial and Primorial Primes' omitted]

7.2: Fermat's Little Theorem(FLT)- FLT and its applications, [Lagrange's alternate proof of Wilson's theorem omitted], inverse of a modulo p using FLT, application-solution of linear congruences [‘ Factors of $2^n + 1$ ‘ omitted], extension of FLT in various directions [‘The Pollard p-1 factoring method’ omitted]

7.4 : Euler's Theorem- motivation, Euler's Phi Function φ, Euler's Theorem, applications, generalisation of Euler's theorem (koshy)

References:	
1	Susanna S Epp: Discrete Mathematics with Applications(4/e) Brooks/ Cole Cengage Learning(2011) ISBN: 978-0-495-39132-6
2	Kenneth H. Rosen: Discrete Mathematics and Its Applications(7/e) McGraw-Hill, NY (2007) ISBN: 978-0-07-338309-5
3	David M. Burton : Elementary Number Theory(7/e) McGraw-Hill (2011) ISBN: 978-0-07-338314-9
4	Gareth A. Jones and J. Mary Jones: Elementary Number Theory, Springer Undergraduate Mathematics Series(1998) ISBN: 978-3-540-76197-6
5	Underwood Dudley :Elementary Number Theory(2/e), Dover Publications (2008) ISBN:978-0-486-46931-7
6	James K Strayer: Elementary Number Theory, Waveland Press, inc. (1994), ISBN:978-1-57766-224-2
7	Kenneth H. Rosen: Elementary Number Theory(6/e), Pearson Education (2018)ISBN: 9780134310053

SECOND SEMESTER

MTS2 B02 CALCULUS OF SINGLE VARIABLE-1

4 hours/week	4 Credits	100 Marks [Int:20+Ext:80]

Aims, Objectives and Outcomes

The mathematics required for *viewing* and analyzing the physical world around us is contained in calculus. While Algebra and Geometry provide us very useful tools for expressing the relationship between static quantities, the concepts necessary to explore the relationship between moving/changing objects are provided in calculus. The objective of the course is to introduce students to the fundamental ideas of limit, continuity and differentiability and also to some basic theorems of *differential calculus*. It is also shown how these ideas can be applied in the problem of sketching of curves and in the solution of some optimization problems of interest in real life. This is done in the first two modules.

The next two modules deal with the other branch of calculus *viz*. *integral calculus.* Historically, it is motivated by the geometric problem of finding out the area of a planar region. The idea of *definite integral* is defined with the notion of limit. A major result is the *Fundamental Theorem of Calculus*, which not only gives a practical way of evaluating the definite integral but establishes the close connection between the two branches of Calculus. The notion of definite integral not only solves the area problem but is useful in finding out the arc length of a plane curve, volume and surface areas of solids and so on. The integral turns out to be a powerful tool in solving problems in physics, chemistry, biology, engineering, economics and other fields. Some of the applications are included in the syllabus.

Syllabus

Text	Calculus: Soo T Tan *Brooks/Cole, Cengage Learning (2010)* *ISBN: 978-0-534-46579-7*

Module-I (20 hrs)

(*Functions and Limits*)

0.2: Functions and their Graphs- Definition of a Function, Describing Functions, Evaluating Functions, Finding the Domain of a Function, The Vertical Line Test, Piecewise Defined Functions, Even and Odd Functions (quick review)

0.4: Combining functions- Arithmetic Operations on Functions, Composition of Functions, Graphs of Transformed Functions, *Vertical Translations, Horizontal Translations, Vertical Stretching and Compressing, Horizontal Stretching and Compressing, Reflecting*

1.1: Intuitive introduction to Limits- A Real-Life Example, Intuitive Definition of a Limit, One-Sided Limits, Using Graphing Utilities to Evaluate Limits

1.2: Techniques for finding Limits- Computing Limits Using the Laws of Limits, Limits of Polynomial and Rational Functions, Limits of Trigonometric Functions, The Squeeze Theorem.

1.3: Precise Definition of a Limit- $\varepsilon - \delta\ definition$, A Geometric Interpretation, Some illustrative examples

1.4: Continuous Functions- Continuity at a Number, Continuity at an Endpoint, Continuity on an Interval, Continuity of Composite Functions, Intermediate Value Theorem

1.5: Tangent Lines and Rate of change- An Intuitive Look, Estimating the Rate of Change of a Function from Its Graph, More Examples Involving Rates of Change, Defining a Tangent Line, Tangent Lines, Secant Lines, and Rates of Change

2.1: The Derivatives- *Definition*, Using the Derivative to Describe the Motion of the Maglev, Differentiation, Using the Graph of f to Sketch the Graph of f' Differentiability, Differentiability and Continuity

2.4: The role of derivative in the real world- Motion Along a Line, Marginal Functions in Economics

2.9: Differentials and Linear Approximations- increments, Differentials, Error Estimates, Linear Approximations, Error in Approximating Δy by dy

Module-II (17 hrs)

(Applications of the Derivative)

3.1: Extrema of Functions -Absolute Extrema of Functions, Relative Extrema of Functions, *Fermat's Theorem*, Finding the Extreme Values of a Continuous Function on a Closed Interval, An Optimization Problem

3.2: The Mean Value Theorem- Rolle's Theorem, *The Mean Value Theorem*, Some Consequences of the Mean Value Theorem, Determining the Number of Zeros of a Function

3.3: Increasing and Decreasing Functions- *definition , inferring the behaviour of function from sign of derivative,* Finding the Relative Extrema of a Function, *first derivative test*

3.4: Concavity and Inflection points- Concavity, Inflection Points, The Second Derivative Test, The Roles of f' and f'' in Determining the Shape of a Graph

3.5: Limits involving Infinity; Asymptotes- Infinite Limits, Vertical Asymptotes, Limits at Infinity, Horizontal Asymptotes, Infinite Limits at Infinity, Precise Definitions

3.6: Curve Sketching-The Graph of a Function, Guide to Curve Sketching, Slant Asymptotes , Finding Relative Extrema Using a Graphing Utility

3.7: Optimization Problems – *guidelines for finding absolute extrema*, Formulating Optimization Problems- *application involving several real life problems*

Module-III (14 hrs)

(Integration)

4.1: Anti derivatives, Indefinite integrals, Basic Rules of Integration, *a few basic integration formulas and rules of integration,* Differential Equations, Initial Value Problems

4.3: Area- An Intuitive Look, The Area Problem, Defining the Area of the Region Under the Graph of a Function-*technique of approximation* *['Sigma Notation' and 'Summation Formulas' Omitted]* An Intuitive Look at Area (Continued), Defining the Area of the Region Under the Graph of a Function-*precise definition*, Area and Distance

4.4: The Definite Integral- Definition of the Definite Integral, Geometric Interpretation of the Definite Integral, The Definite Integral and Displacement, Properties of the Definite Integral , More General Definition of the Definite Integral

4.5: The Fundamental Theorem of Calculus- How Are Differentiation and Integration Related?, The Mean Value Theorem for Definite Integrals, The Fundamental Theorem of Calculus: Part I, *inverse relationship between differentiation and integration,* Fundamental Theorem of Calculus: Part 2, Evaluating Definite Integrals Using Substitution, Definite Integrals of Odd and Even Functions, The Definite Integral as a Measure of Net Change

Module-IV (13 hrs)

(Applications of Definite Integral)

5.1: Areas between Curves- A Real Life Interpretation, The Area Between Two Curves, Integrating with Respect to *y-adapting to the shape of the region*, What Happens When the Curves Intertwine?

5.2: Volume – Solids of revolution, *Volume by* Disk Method, *Region revolved about the x-axis, Region revolved about the y-axis , Volume by the* Method of Cross Sections *[' Washer Method' omitted]*

5.4: Arc Length and Areas of surfaces of revolution- Definition of Arc Length, Length of a Smooth Curve, *arc length formula*, The Arc Length Function, *arc length differentials*, Surfaces of Revolution, *surface area as surface of revolution,*

5.5: Work-Work Done by a Constant Force, Work Done by a Variable Force, Hook's Law, Moving non rigid matter, Work done by an expanding gas

5.7: Moments and Center of Mass- Measures of Mass, Center of Mass of a System on a Line, Center of Mass of a System in the Plane, Center of Mass of Laminas *[upto and including Example 3; rest of the section omitted]*

References:

1	Joel Hass, Christopher Heil & Maurice D. Weir : Thomas' Calculus(14/e) *Pearson (2018) ISBN 0134438981*
2	Robert A Adams & Christopher Essex : Calculus *Single Variable (8/e) Pearson Education Canada (2013) ISBN:* 0321877403
3	Jon Rogawski & Colin Adams : Calculus *Early Transcendentals* (3/e) *W. H. Freeman and Company(2015) ISBN: 1319116450*
4	Anton, Bivens & Davis : Calculus *Early Transcendentals* (11/e) *John Wiley & Sons, Inc.(2016) ISBN: 1118883764*
5	James Stewart : Calculus (8/e) *Brooks/Cole Cengage Learning(2016) ISBN: 978-1-285-74062-1*
6	Jerrold Marsden & Alan Weinstein : Calculus I and II (2/e*) Springer Verlag NY (1985) ISBN 0-387-90974-5 : ISBN 0-387-90975-3*

THIRD SEMESTER

MTS3 B03 CALCULUS OF SINGLE VARIABLE-2

5 hours/week 4 Credits 100 Marks [Int:20+Ext:80]

Aims, Objectives and Outcomes

Using the idea of definite integral developed in previous semester, the natural logarithm function is defined and its properties are examined. This allows us to define its inverse function namely the *natural exponential function* and also the *general exponential function*. Exponential functions model a wide variety of phenomenon of interest in science, engineering, mathematics and economics. They arise naturally when we model the growth of a biological population, the spread of a disease, the radioactive decay of atoms, and the study of heat transfer problems and so on. We also consider certain combinations of exponential functions namely *hyperbolic functions* that also arise very frequently in applications such as the study of shapes of cables hanging under their own weight.

After this, the students are introduced to the idea of *improper integrals*, their convergence and evaluation. This enables to study a related notion of convergence of a *series*, which is practically done by applying several different tests such as integral test, comparison test and so on. As a special case, a study on power series- their region of convergence, differentiation and integration etc.,- is also done.

A detailed study of plane and space curves is then taken up. The students get the idea of parametrization of curves, they learn how to calculate the arc length, curvature etc. using parametrization and also the area of surface of revolution of a parametrized plane curve. Students are introduced into other coordinate systems which often simplify the equation of curves and surfaces and the relationship between various coordinate systems are also taught. This enables them to directly calculate the arc length and surface areas of revolution of a curve whose equation is in polar form. At the end of the course, the students will be able to handle *vectors* in dealing with the problems involving geometry of lines, curves, planes and surfaces in space and have acquired the ability to sketch curves in plane and space given in vector valued form.

Syllabus

Text	Calculus: Soo T Tan *Brooks/Cole, Cengage Learning (2010)* *ISBN: 978-0-534-46579-7*

Module-I (20 hrs)

(The Transcendental Functions)

6.1: The Natural logarithmic function- *definition*, The Derivative of $ln\ x$, Laws of Logarithms, The Graph of the Natural Logarithmic Function, The Derivatives of Logarithmic Functions, Logarithmic Differentiation, Integration Involving Logarithmic Functions

6.2: Inverse Functions-The Inverse of a Function, The Graphs of Inverse Functions, Which Functions have Inverses?, Finding the Inverse of a Function, Continuity and Differentiability of Inverse Functions.

6.3: Exponential Functions- The number e, Defining the Natural Exponential Function, *properties*, The Laws of Exponents, The Derivatives of Exponential Functions, Integration of the Natural Exponential Function

6.4: General Exponential and Logarithmic Functions - Exponential Functions with Base a, *laws of exponents*, The Derivatives of a^x,a^u, Graphs of y= a^x, integrating a^x , Logarithmic Functions with Base a, *change of base formula*, The Power Rule (General Form), The Derivatives of Logarithmic Functions with Base a, The Definition of the Number e as a Limit *['Compound Interest' omitted]*

6.5: Inverse trigonometric functions- *definition, graph, inverse properties*, Derivative of inverse trigonometric functions, Integration Involving Inverse Trigonometric Functions

6.6: Hyperbolic functions- The Graphs of the Hyperbolic Functions, Hyperbolic Identities, Derivatives and Integrals of Hyperbolic Functions, Inverse Hyperbolic Functions, *representation in terms of logarithmic function*, Derivatives of Inverse Hyperbolic Functions, An Application

6.7: Indeterminate forms and l'Hôpital rule- *motivation*, The Indeterminate Forms $\frac{0}{0}$ and $\frac{\infty}{\infty}$, The Indeterminate Forms $\infty - \infty$ and $0.\infty$, The Indeterminate Forms $0^0, \infty^0$ *and* 1^∞

Module-II (20 hrs)

(Infinite Sequences and Series)

7.6: Improper integrals – *definition*, Infinite Intervals of Integration, Improper Integrals with Infinite Discontinuities, A Comparison Test for Improper Integrals

9.1: Sequences- *definition, recursive definition,* Limit of a Sequence, *limit laws, squeeze theorem*, Bounded Monotonic Sequences, *definition, monotone convergence theorem (only statement; its proof omitted)*

9.2: Series- *defining the sum*, *convergence and divergence*, Geometric Series, The Harmonic Series, The Divergence Test, Properties of Convergent Series

9.3: The Integral Test – investigation of convergence ,integral test, The p-Series, *its convergence and divergence*

9.4: The Comparison Test- *test series*, The Comparison Test, The Limit Comparison Test

9.5: Alternating Series- *definition, the alternating series test, its proof, examples,* Approximating the Sum of an Alternating Series by S_n

9.6: Absolute Convergence- *definition*, *conditionally convergent*, The Ratio Test, The Root Test, Summary of Tests for Convergence and Divergence of Series, Rearrangement of Series

Module-III (20 hrs)

9.7: Power Series- *definition*, Interval of Convergence, *radius of convergence*, Differentiation and Integration of Power Series

9.8: Taylor and Maclaurin Series- *definition, Taylor and Maclaurin series of functions,* Techniques for Finding Taylor Series

10.2: Plane Curves and Parametric Equations- Why We Use Parametric Equations, Sketching Curves Defined by Parametric Equations

10.3: The Calculus of parametric equations- Tangent Lines to Curves Defined by Parametric Equations, Horizontal and Vertical Tangents, Finding $\frac{d^2y}{dx^2}$ from Parametric Equations, The Length of a Smooth Curve, The Area of a Surface of Revolution

10.4: Polar coordinate-The Polar Coordinate System, Relationship Between Polar and Rectangular Coordinates, Graphs of Polar Equations, Symmetry, Tangent Lines to Graphs of Polar Equations

10.5:Areas and Arc Lengths in polar coordinates-Areas in Polar Coordinates, *area bounded by polar curves*, Area Bounded by Two Graphs, Arc Length in Polar Coordinates, Area of a Surface of Revolution, Points of Intersection of Graphs in Polar Coordinates

Module-IV	**(20 hrs)**

11.5 : Lines and Planes in Space-Equations of Lines in Space, *parametric equation, symmetric equation of a line,* Equations of Planes in Space, *standard equation,* Parallel and Orthogonal Planes, The Angle Between Two Planes, The Distance Between a Point and a Plane

11.6: Surfaces in Space- Traces, Cylinders, Quadric Surfaces, *Ellipsoids, Hyperboloids of One Sheet, Hyperboloids of Two Sheets, Cones, Paraboloids, Hyperbolic Paraboloids*

11.7: Cylindrical and Spherical Coordinates-The Cylindrical Coordinate System, *converting cylindrical to rectangular and vice verse*, The Spherical Coordinate System, *converting spherical to rectangular and vice verse,*

12.1: Vector Valued functions and Space Curves- *definition of vector function*, Curves Defined by Vector Functions, *['Example 7' omitted]* Limits and Continuity

12.2:Differentiation and Integration of Vector-Valued Function- The Derivative of a Vector Function, Higher-Order Derivatives, Rules of Differentiation, Integration of Vector Functions,

12.3: Arc length and Curvature- Arc Length *of a space curve*, Smooth Curves, Arc Length Parameter, *arc length function,* Curvature, *formula for finding curvature,* Radius of Curvature,

12.4: Velocity and Acceleration- Velocity, Acceleration, and Speed; Motion of a Projectile

12.5: Tangential and Normal Components of Acceleration- The Unit Normal, *principal unit normal vector*, Tangential and Normal Components of Acceleration *[The subsections ' Kepler's Laws of Planetary Motion ', and ' Derivation of Kepler's First Law' omitted]*

References:	
1	Joel Hass, Christopher Heil & Maurice D. Weir : Thomas' Calculus (14/e) *Pearson(2018) ISBN 0134438981*
2	Robert A Adams & Christopher Essex : Calculus *Single Variable (8/e) Pearson Education Canada (2013) ISBN:* 0321877403
3	Jon Rogawski & Colin Adams : Calculus *Early Transcendentals* (3/e) W. H. Freeman and Company(2015) *ISBN: 1319116450*
4	Anton, Bivens & Davis : Calculus *Early Transcendentals* (11/e) *John Wiley & Sons, Inc.(2016) ISBN: 1118883764*
5	James Stewart : Calculus (8/e) *Brooks/Cole Cengage Learning(2016) ISBN: 978-1-285-74062-1*
6	Jerrold Marsden & Alan Weinstein : Calculus I and II (2/e) *Springer Verlag NY(1985) ISBN 0-387-90974-5 : ISBN 0-387-90975-3*

FOURTH SEMESTER

MTS4 B04 LINEAR ALGEBRA

5 hours/week **4 Credits** **100 Marks [Int:20+Ext:80]**

Aims, Objectives and Outcomes

An introductory treatment of linear algebra with an aim to present the fundamentals in the clearest possible way is intended here. Linear algebra is the study of linear systems of equations, vector spaces, and linear transformations. Virtually every area of mathematics relies on or extends the tools of linear algebra. Solving systems of linear equations is a basic tool of many mathematical procedures used for solving problems in science and engineering. A number of methods for solving a system of linear equations are discussed. In this process, the student will become competent to perform matrix algebra and also to calculate the inverse and determinant of a matrix. Another advantage is that the student will come to understand the modern view of a matrix as a linear transformation. The discussion necessitates the introduction of central topic of linear algebra namely the concept of a *vector space*. The familiarity of the students with planar vectors and their algebraic properties under vector addition and scalar multiplication will make them realize that the idea of a general vector space is in fact an *abstraction* of what they already know. Several examples and general properties of vector spaces are studied. The idea of a subspace, spanning vectors, basis and dimension are discussed and fundamental results in these areas are explored. This enables the student to understand the relationship among the solutions of a given system of linear equations and some important subspaces associated with the coefficient matrix of the system.

After this, some basic matrix transformations in the vector spaces $\mathbb{R}^2$ *and* $\mathbb{R}^3$,having interest in the field of computer graphics, engineering and physics are studied by specially pinpointing to their geometric effect.

Just like choosing an appropriate coordinate system greatly simplifies a problem at our hand as we usually see in analytic geometry and calculus, a right choice of the basis of the vector space $\mathbb{R}^n$ greatly simplifies the analysis of a matrix operator on it. With this aim in mind, a study on eigenvalues and eigenvectors of a given matrix (*equivalently*, that of the corresponding matrix operator) is taken up. Practical method of finding out the eigenvalues from the characteristic equation and the corresponding eigenvectors are also discussed. A bonus point achieved during this process is a test for the invertibility of a square matrix. As diagonal matrices are the matrices with simplest structure, the idea of *diagonalization* of a matrix (and hence the diagonalization of a matrix operator) is introduced and students learn a few fundamental results involving diagonalization and eigenvalues which enable them to check whether diagonalization is possible. They realise that there are matrices that cannot be diagonalized and even learn to check it. Also they are taught a well defined procedure for diagonalizing a given matrix, if this is actually the case. The topic is progressed further to obtain the ultimate goal of *spectral decomposition* of a symmetric matrix. In this process , students realise that every symmetric matrix is diagonalizable and that this diagonalization can be done in a special way ie., by choosing an *orthogonal matrix* to perform the diagonalization. This is known as orthogonal diagonalization. Students also learn that *only* symmetric matrices with real entries can be orthogonally diagonalized and using Gram-Schmidt process a well defined procedure for writing such a diagonalization is also taught. In short, the course gives the students an opportunity to learn the fundamentals of linear algebra by capturing the ideas geometrically, by justifying them algebraically and by preparing them to apply it in several different fields such as data communication, computer graphics, modelling etc.

Syllabus

Text	Elementary Linear Algebra: Application Version(11/e) :Howard Anton & Chris Rorres *Wiley(2014) ISBN 978-1-118-43441-3*

Module-I (17 hrs)

Systems of Linear Equations & Matrices :-

1.1: Introduction to Systems of Linear Equations- *linear equation in n variables, linear system of m equations in n variables, solution*, Linear Systems in Two and Three Unknowns, *solution by geometric analysis*, *consistent and inconsistent systems, linear system with no, one, and infinite number of solutions*, augmented matrix and elementary row operations

1.2: Gaussian elimination - Considerations in Solving Linear Systems, Echelon Forms, *reduced row echelon form*, Elimination Methods, *Gauss–Jordan elimination*, *Gaussian elimination*, Homogeneous Linear Systems, *Free Variables, Free Variable Theorem for Homogeneous Systems*, Gaussian Elimination andBack- Substitution, Some Facts about Echelon Forms

1.3: Matrices and Matrix operations- Matrix Notation and Terminology, *row vector , column vector , square matrix of order n , Operations on Matrices* , Partitioned Matrices, Matrix Multiplication by Columns and by Rows, Matrix Products as Linear Combinations, *linear combination* of column vectors, Column-Row Expansion, Matrix Form of a Linear System, Transpose of a Matrix, Trace of a Matrix

1.4: Inverses and algebraic properties of matrices- *Properties of Matrix Addition and Scalar Multiplication, Properties of Matrix Multiplication, Zero Matrices and Properties, Identity Matrices, Inverse of a Matrix, Properties of Inverses, Solution of a Linear System by Matrix Inversion*, Powers of a Matrix , Matrix Polynomials, *Properties of the Transpose*

1.5: Elementary matrices and a method for finding A^{-1}-*row equivalence, elementary matrix, Row Operations by Matrix Multiplication, invertibility of* *elementary matrices, invertibility and equivalent statements,* A Method for Inverting Matrices, *Inversion Algorithm, illustrations.*

1.6: More on linear systems and invertible matrices - Number of Solutions of a Linear System, Solving Linear Systems by Matrix Inversion, *Linear Systems with a Common Coefficient Matrix, Properties of Invertible Matrices, equivalent statements for unique solution of* $Ax = b$*, determining consistency*

1.7: Diagonal, Triangular and Symmetric matrices-*Diagonal Matrices, Inverses and Powers of Diagonal Matrices, Triangular Matrices. Properties of Triangular Matrices, Symmetric Matrices, algebraic properties of symmetric matrices, Invertibility of Symmetric Matrices*

1.8: Matrix transformation- *definition*, Properties of Matrix Transformations, *standard matrix*, A Procedure for Finding Standard Matrices

2.1: Determinants by cofactor expansion- *minors, cofactors, cofactor expansion, Definition of a General Determinant, A Useful Technique for Evaluating 2 × 2 and 3 × 3 Determinants*

2.2: Evaluating determinants by row reduction- a *few basic theorems, elementary row operations and determinant, determinant of elementary matrices, determinant by row reduction*

Module-II	**(18 hrs)**

General Vector Spaces

4.1: Real vector space - *Vector Space Axioms, examples, Some Properties of Vectors*

4.2: Subspaces- *definition, criteria for a subset to be a subspace*, *examples, Building Subspaces, linear combination, spanning, Solution Spaces of Homogeneous Systems as subspace, The Linear Transformation Viewpoint , kernel, different set of vectors spanning the subspace.*

4.3: Linear Independence- *Linear Independence and Dependence, illustrations , A Geometric Interpretation of Linear Independence, Wronskian, linear independence using wronskian*

4.4: Coordinates and basis-*Coordinate Systems in Linear Algebra, Basis for a Vector Space, finite and infinite dimensional vector spaces, illustrations, Coordinates Relative to a Basis, Uniqueness of Basis Representation*

4.5: Dimension- *Number of Vectors in a Basis* , dimension, *Some Fundamental Theorems, dimension of subspaces,*

Module-III (22 hrs)

4.6: Change of basis -*Coordinate Maps, Change of Basis, Transition Matrices, Invertibility of Transition Matrices, An Efficient Method for Computing Transition Matrices for* $\mathbb{R}^n$*, Transition to the Standard Basis for* $\mathbb{R}^n$

4.7: Row space, Column space and Null space- vector spaces associated with matrices, consistency of linear system, *Bases for Row Spaces, Column Spaces, and Null Spaces, basis from* row echelon form, *Basis for the Column Space of a Matrix, row equivalent matrices and relationship between basis for column space, Bases Formed from Row and Column Vectors of a Matrix*

4.8: Rank Nullity and Fundamental matrix spaces- equality of dimensions of row and column spaces, *Rank and Nullity,* Dimension Theorem for Matrices**,** *The Fundamental Spaces of a Matrix, rank of a matrix and its transpose, A Geometric Link Between the Fundamental Spaces, orthogonal complement,, invertibility and equivalent statements, Applications of Rank, Overdetermined and Underdetermined Systems*

4.9: Basic matrix transformations in R^2 and R^3 -*Reflection Operators, Projection Operators, Rotation Operators, Rotations in* $\mathbb{R}^3$ *, Dilations and Contractions, Expansions and Compressions, Shears, Orthogonal Projections onto LinesThrough the Origin, Reflections About Lines Through the Origin*

4.10: Properties of matrix transformation- *Compositions of Matrix Transformations, One-to-One Matrix Transformations, Kernel and Range, fundamental relationship between invertibility of a matrix and its matrix transformation, Inverse of a One-to-One Matrix Operator*

Module-IV (23 hrs)

4.11: Geometry of matrix operators-*Transformations of Regions, Images of Lines Under Matrix Operators, Geometry of Invertible Matrix Operators, Elementary matrix and its matrix transformation, consequence*

5.1: Eigen values and Eigen Vectors- *definition, Computing Eigenvalues and Eigenvectors, characteristic equation, alternative ways of describing eigen values, Finding Eigenvectors and Bases for Eigenspaces, Eigenvalues and Invertibility, Eigenvalues of General Linear Transformations,*

5.2: Diagonalization-*The Matrix Diagonalization Problem, linear independence of eigen vectors and diagonalizability, Procedure for Diagonalizing a Matrix,* *Eigenvalues of Powers of a Matrix, Computing Powers of a Matrix, Geometric and Algebraic Multiplicity*

6.1: Inner Product – *definition of* General inner product, *Euclidean inner product (or the standard inner product) on* $\mathbb{R}^n$, *norm of a vector, properties (upto and including theorem 6.1.1), a few examples (only example7 and example 10) [rest of the section omitted]*

6.2: Angle and orthogonality in Inner product spaces- *only the definition of orthogonality in a real inner product space (to be motivated by the relation in the definition (3) of section 3.2) and examples(2),(3) and (4)*

6.3: Gram–Schmidt Process- *definition of* Orthogonal and Orthonormal Sets, *examples,linear independence of orthogonal set, orthonormal basis,* Coordinates Relative to Orthonormal Bases *[‘Orthogonal Projections’ omitted]* The Gram–Schmidt Process *[only statement of Theorem 6.3.5 and the step by step construction technique are required; derivation omitted], illustrations-* examples 8 and 9, Extending Orthonormal Sets to Orthonormal Bases *[rest of the section omitted]*

7.1: Orthogonal Matrices- *definition, characterisation of orthogonal matrices, properties of orthogonal matrices,* Orthogonal Matrices as Linear Operators, *a geometric interpretation [rest of the section omitted]*

7.2: Orthogonal Diagonalization- The Orthogonal Diagonalization Problem, Conditions for Orthogonal Diagonalizability, Properties of Symmetric Matrices, *Procedure for Orthogonally Diagonalizing an n × n Symmetric Matrix*, Spectral Decomposition *(upto and including example2) [rest of the section omitted]*

References:

1	Jim DeFranza, Daniel Gagliardi: Introduction to Linear Algebra with Applications *Waveland Press, Inc(2015)ISBN: 1-4786-2777-8*
2	Otto Bretscher: Linear Algebra with Applications(5/e) Pearson Education, Inc (2013) ISBN: 0-321-79697-7
3	Ron Larson, Edwards, David C Falvo : Elementary Linear Algebra(6/e) *Houghton Mifflin Harcourt Publishing Company(2009) ISBN: 0-618-78376-8*
4	David C. Lay, Steven R. Lay, Judi J. McDonald: Linear Algebra and its Application (5/e) *Pearson Education, Inc(2016) ISBN: 0-321-98238-X*
5	Martin Anthony, Michele Harvey: Linear Algebra: Concepts and Methods *Cambridge University Press(2012) ISBN: 978-0-521-27948-2*
6	Jeffrey Holt: Linear Algebra with Applications *W. H. Freeman and Company (2013) ISBN: 0-7167-8667-2*

MTS5 B05 ABSTRACT ALGEBRA

5 hours/week | **4 Credits** | **100 Marks Int:20+Ext:80]**

Aims, Objectives and Outcomes	

The brilliant mathematician Evariste Galois developed an entire theory that connected the solvability by radicals of a polynomial equation with the *permutation group* of its roots. The theory now known as *Galois theory* solves the famous problem of *insolvability of quintic*. A study on *symmetric functions* now becomes inevitable. One can now observe the connection emerging between classical algebra and modern algebra. The last three modules are therefore devoted to the discussion on basic ideas and results of abstract algebra. Students understand the abstract notion of a group, learn several examples, are taught to check whether an *algebraic system* forms a group or not and are introduced to some fundamental results of group theory. The idea of structural similarity, the notion of cyclic group, permutation group , various examples and very fundamental results in the areas are also explored.

Syllabus

Text Abstract Algebra(3/e): John A Beachy and William D Blair Waveland Press, Inc.(2006) ISBN: 1-57766-443-4 .

Module- I (15 hrs)

1.4: Integers modulo n - congruence class modulo n, addition and multiplication, divisor of zero, multiplicative inverse

2.2: Equivalence relations-basic idea, definition, equivalence class, factor set, partition and equivalence relation, examples and illustrations

2.3: Permutations- definition, cycles, product of cycles, permutation as product of disjoint cycles, order of cycles, transposition, even and odd transpositions

Module- II (25 hrs)

3.1: Definition of Group-binary operation, uniqueness of identity and inverse, definition and examples of groups, properties, Abelian group, finite and infinite groups, general linear groups

3.2: Subgroups-the notion of subgroup, examples, conditions for a subgroup, cyclic subgroups, order of an element, Lagrange theorem, Euler's theorem

3.3: Constructing examples- groups with order upto 6, multiplication table, product of subgroups, direct products, Klein four group as direct product, subgroup generated by a subset

3.4: Isomorphism – definition, consequences, structural properties, method of showing that groups are not isomorphic, isomorphic and non isomorphic groups.

Module- III (25 hrs)

3.5: Cyclic groups- subgroups of cyclic groups, characterisation, generators of a finite cyclic group, structure theorem for finite cyclic group, exponent of a group, characterisation of cyclic groups among finite abelian groups.

3.6: Permutation groups- definition, Cayley's theorem, rigid motions of n-gons, dihedral group, alternating group

3.7: Homomorphism - basic idea, examples, definition, properties, kernel, normal subgroups, subgroups related via homomorphism

3.8: Cosets- left and right cosets, normal subgroups and factor groups, fundamental homomorphism theorem, simple groups, examples and illustrations of concepts

Module- IV (15 hrs)

7.1: (Structure of Groups) Isomorphism theorems; Automorphism- first isomorphism theorem, second isomorphism theorem, inner automorphism

5.1: Commutative Rings ; Integral Domains- definition, examples, subring, criteria to be a subring, divisor of zero, integral domain, finite integral domain.

References:	
1	Joseph A. Gallian : Contemporary Abstract Algebra(9/e) *Cengage Learning, Boston(2017) ISBN: 978-1-305-65796-0*
2	John B Fraleigh : A First Course in Abstract Algebra(7/e) *Pearson Education LPE (2003) ISBN 978-81-7758-900-9*
3	David Steven Dummit, Richard M. Foote: Abstract Algebra(3/e) *Wiley, (2004) ISBN: 8126532289*
4	Linda Gilbert and Jimmie Gilbert: Elements of Modern Algebra (8/e) *Cengage Learning, Stamford(2015) ISBN: 1-285-46323-4*
5	John R. Durbin : Modern Algebra: An Introduction(6/e) *Wiley(2015) ISBN: 1118117611*
6	Jeffrey Bergen: A Concrete Approach to Abstract Algebra- From the integers to Insolvability of Quintic *Academic Pres [Elsever](2010)ISBN: 978-0- 12-374941-3*

MTS5 B06 BASIC ANALYSIS

5 hours/week	4 Credits	100 Marks Int:20+Ext:80]

Aims, Objectives and Outcomes	

In this course, basic ideas and methods of real and complex analysis are taught. Real analysis is a theoretical version of single variable calculus. So many familiar concepts of calculus are reintroduced but at a much deeper and more rigorous level than in a calculus course. At the same time there are concepts and results that are new and not studied in the calculus course but very much needed in more advanced courses. The aim is to provide students with a level of mathematical sophistication that will prepare them for further work in mathematical analysis and other fields of knowledge, and also to develop their ability to analyse and prove statements of mathematics using logical arguments. The course will enable the students

- to learn and deduce rigorously many properties of real number system by assuming a few fundamental facts about it as axioms. In particular they will learn to prove Archimedean property, density theorem, existence of a positive square root for positive numbers and so on and the learning will help them to appreciate the beauty of logical arguments and embolden them to apply it in similar and unknown problems.

- to know about sequences ,their limits, several basic and important theorems involving sequences and their applications . For example, they will learn how *monotone convergence theorem* can be used in establishing the divergence of the *harmonic series*, how it helps in the calculation of square root of positive numbers and how it establishes the existence of the *transcendental* number *e (Euler constant).*

- to understand some basic topological properties of real number system such as the concept of open and closed sets, their properties, their characterization and so on.

- to get a rigorous introduction to algebraic, geometric and topological structures of complex number system, functions of complex variable, their limit and continuity and so on. Rich use of geometry, comparison between real and complex calculus-areas where they agree and where they differ, the study of mapping properties of a few important complex functions exploring the underlying geometry etc. will demystify student's belief that complex variable theory is incomprehensible.

Syllabus

Text (1) Introduction to Real Analysis(4/e) : Robert G Bartle, Donald R Sherbert John Wiley & Sons (2011) ISBN 978-0-471-43331-6

Text (2) Complex Analysis A First Course with Applications (3/e): Dennis Zill & Patric Shanahan Jones and Bartlett Learning(2015) ISBN:1-4496-9461-6

Module- I Text (1) (20 hrs)

1.3: Finite and Infinite Sets-definition, countable sets, denumerability of ℚ, union of countable sets, cantor's theorem

2.1: The Algebraic and Order Properties of ℝ - algebraic properties, basic results, rational and irrational numbers, irrationality of √2, Order properties, arithmetic-geometric inequality, Bernoulli's Inequality

2.2: Absolute Value and the Real Line- definition, basic results, Triangle Inequality, The real line, ε-neighborhood

2.3: The Completeness Property of ℝ-Suprema and Infima, alternate formulations for the supremum, The Completeness Property

Module- II Text (1) (21 hrs)

2.4: Applications of the Supremum Property- The Archimedean Property, various consequences, Existence of √2, Density of Rational Numbers in ℝ, The Density Theorem, density of irrationals

2.5: Intervals-definition, Characterization of Intervals, Nested Intervals, Nested Intervals Property, The Uncountability of ℝ, [binary, decimal and periodic representations omitted] Cantor's Second Proof.

3.1: Sequences and Their Limits- definitions, convergent and divergent sequences, Tails of Sequences, Examples

3.2: Limit Theorems- sum, difference, product and quotients of sequences, Squeeze Theorem, ratio test for convergence

3.3: Monotone Sequences-definition, monotone convergence theorem, divergence of harmonic series, calculation of square root, Euler's number

Module- III	Text (1)	(18 hrs)

3.4: Subsequences and the Bolzano-Weierstrass Theorem- definition, limit of subsequences, divergence criteria using subsequence, The Existence of Monotone Subsequences, monotone subsequence theorem, The Bolzano-Weierstrass Theorem, Limit Superior and Limit Inferior

3.5: The Cauchy Criterion- Cauchy sequence, Cauchy Convergence Criterion, applications, contractive sequence

3.6: Properly divergent sequences-definition, examples, properly divergent monotone sequences, ‘‘comparison theorem’’, ‘‘limit comparison theorem’’

11.1: Open and Closed sets in $\mathbb{R}$, neighborhood, open sets, closed sets, open set properties, closed set properties, Characterization of Closed Sets, cluster point, Characterization of Open Sets, The Cantor Set, properties

Module- IV	Text (2)	(21 hrs)

1.1: Complex numbers and their properties- *definition*, arithmetic operations, conjugate, inverses, reciprocal

1.2: Complex Plane- vector representation, modulus, properties, triangle inequality

1.3: Polar form of complex numbers- polar representation, principal argument, multiplication and division, argument of product and quotient, integer powers, de Moivre’s formula.

1.4: Powers and roots- roots, principal n^{th} root

1.5: Sets of points in the complex plane- circles, disks and neighbourhoods, open sets, annulus, domains, regions, bounded sets

2.1: Complex Functions- definition, real and imaginary parts of complex function, complex exponential function, exponential form of a complex number, Polar Coordinates

2.2: Complex Functions as mappings- complex mapping, illustrations, Parametric curves in complex planes, common parametric curves, image of parametric curves under complex mapping [The subsection ‘Use of Computers’ omitted]

2.3: Linear Mappings- Translations, Rotations, Magnifications, general linear mapping, image of geometric shapes under linear map.

2.4: Special Power functions- The power function z^n, The power function $z^{(1/n)}$, principal square root function, Inverse Functions, multiple valued functions

References:	
1	*Charles G. Denlinger: Elements of Real Analysis Jones and Bartlett Publishers Sudbury, Massachusetts (2011) ISBN:0-7637-7947-4 [Indian edition: ISBN- 9380853157]*
2	*David Alexander Brannan: A First Course in Mathematical Analysis Cambridge University Press,US(2006) ISBN: 9780521684248*
3	*John M. Howie: Real Analysis Springer Science & Business Media(2012) [Springer Undergraduate Mathematics Series] ISBN: 1447103416*
4	*James S. Howland: Basic Real Analysis Jones and Bartlett Publishers Sudbury, Massachusetts (2010) ISBN:0-7637-7318-2*
5	*James Ward Brown, Ruel Vance Churchill: Complex variables and applications(8/e) McGraw-Hill Higher Education, (2009) ISBN: 0073051942*
6	*Alan Jeffrey: Complex Analysis and Applications(2/e) Chapman and Hall/CRC Taylor Francis Group(2006)ISBN:978-1-58488-553-5*
7	Saminathan Ponnusamy, Herb Silverman: Complex Variables with Applications Birkhauser Boston(2006) ISBN:0-8176-4457-4
8	Terence Tao: Analysis I & II (3/e) TRIM 37 & 38 Springer Science+Business Media Singapore 2016; Hindustan book agency(2015) ISBN 978-981-10-1789-6 (eBook) & ISBN 978-981-10-1804-6 (eBook)
9	Ajith Kumar & S Kumaresan : A Basic Course in Real Analysis CRC Press, Taylor & Francis Group(2014) ISBN: 978-1-4822-1638-7 (eBook – PDF)
10	Hugo D Junghenn : A Course in Real Analysis CRC Press, Taylor & Francis Group (2015) ISBN: 978-1-4822-1928-9 (eBook - PDF)

FIFTH SEMESTER

MTS5 B07 NUMERICAL ANALYSIS

4 hours/week	3 Credits	75 Marks [Int:15+Ext:60]

Aims, Objectives and Outcomes

The goal of numerical analysis is to provide techniques and algorithms to find *approximate numerical solution* to problems in several areas of mathematics where it is impossible or hard to find the actual/closed form solution by analytical methods and also to make an *error analysis* to ascertain the accuracy of the *approximate solution.* The subject addresses a variety of questions ranging from the approximation of functions and integrals to the approximate solution of algebraic, transcendental, differential and integral equations, with particular emphasis on the stability, accuracy, efficiency and reliability of numerical algorithms. The course enables the students to

- Understand several methods such as bisection method, fixed point iteration method, regula falsi method etc. to find out the approximate numerical solutions of algebraic and transcendental equations with desired accuracy.
- Understand the concept of interpolation and also learn some well known interpolation techniques.
- Understand a few techniques for numerical differentiation and integration and also realize their merits and demerits.
- Find out numerical approximations to solutions of initial value problems and also to understand the efficiency of various methods.

Syllabus

Text	Numerical Analysis (10/e): *Richard L. Burden, J Douglas Faires, Annette M. Burden Brooks Cole Cengage Learning(2016) ISBN:978-1-305-25366-7*

Module-I (28 hrs)

Solutions of Equations in One Variable

Note: Students should be familiar with concepts and definitions such as 'round off error', rate of convergence ' etc. discussed in sections 1.2 and 1.3

Introduction

2.1: The Bisection Method

2.2: Fixed-Point Iteration

2.3: Newton's Method and Its Extensions- *Newton's Method (Newton-Raphson method), Convergence using Newton's Method, The Secant Method, The Method of False Position*

2.4: Error Analysis for Iterative Methods- Order of Convergence, *linear and quadratic convergence,* Multiple Roots, *Modified Newton's method for faster convergence*

[Algorithms are omitted]

Interpolation and Polynomial Approximation

Introduction

3.1: Interpolation and the Lagrange Polynomial- *motivation*, Lagrange Interpolating Polynomials, *error bound*

3.2: Data Approximation and Neville's Method- *motivation*, Neville's Method, *recursive method to generate Lagrange polynomial approximations*.

3.3: Divided Differences- k^{th} *divided difference, Newton's divided difference formula,* Forward Differences, Newton Forward-Difference Formula, Backward Differences, Newton Backward–Difference Formula, Centered Differences, *Stirling's formula.*

[Algorithms are omitted]

Module-II (18 hrs)

Numerical Differentiation and Integration

Introduction

4.1: Numerical Differentiation- *approximation of first derivative by forward difference formula, backward difference formula, Three-Point Formulas, Three-Point Endpoint Formula, Three-Point Midpoint Formula [Five-Point Formulas, Five-Point Endpoint Formula, Five-Point Midpoint Formula omitted]* Second Derivative Midpoint Formula *to approximate second derivative,* Round-Off Error Instability

4.3:Elements of Numerical Integration-*numerical quadrature,* The Trapezoidal Rule, Simpson's Rule, Measuring Precision, Closed Newton-Cotes Formulas, Simpson's Three-Eighths rule, Open Newton-Cotes Formulas

4.4: Composite Numerical Integration-*composite Simpson's rule, composite trapezoidal rule, composite midpoint rule, round off error stability*

4.7:Gaussian Quadrature-*motivation*, Legendre Polynomial, Gaussian Quadrature on Arbitrary Intervals
[Algorithms are omitted]

Module-III (18 hrs)

Initial-Value Problems for Ordinary Differential Equations

Introduction

5.1 The Elementary Theory of Initial-Value Problems

5.2: Euler's Method- *derivation using Taylor formula,* Error bounds for Euler Method

5.3: Higher-Order Taylor Methods- *local truncation error*, *Taylor method of order n and order of local truncation error*

5.4: Runge-Kutta Methods- *only* Mid Point Method, Modified Euler's Method and Runge-Kutta Method of Order Four are required. *[derivation of formula omitted in each case]*

5.6: Multistep Methods- *basic idea, definition*, Adams-Bashforth Two-Step Explicit Method, Adams-Bashforth Three-Step Explicit Method, Adams-Bashforth Four-Step Explicit Method, Adams-Moulton Two-Step Implicit Method, Adams-Moulton Three-Step Implicit Method, Adams-Moulton

Four-Step Implicit Method, Predictor-Corrector Methods *[derivation of formula omitted in each case]* [Algorithms are omitted]

References:

1	Kendall E. Atkinson, Weimin Han: Elementary Numerical Analysis(3/e) *John Wiley & Sons(2004) ISBN:0-471-43337-3[Indian Edition by Wiley India ISBN: 978-81-265-0802-0]*
2	James F. Epperson: An Introduction to Numerical Methods and Analysis(2/e) *John Wiley & Sons(2013)ISBN: 978-1-118-36759-9*
3	Timothy Sauer: Numerical Analysis(2/e) *Pearson (2012) ISBN: 0-321-78367-0*
4	S S Sastri : Introductory Methods of Numerical Analysis(5/e) *PHI Learning Pvt. Ltd.(2012) ISBN:978-81-203-4592-8*
5	Ward Cheney,David Kincaid : Numerical Mathematics and Computing (6/e) *Thomson Brooks/Cole(2008) ISBN: 495-11475-8*

FIFTH SEMESTER

MTS5 B08 LINEAR PROGRAMMING

3 hours/week **3 Credits** **75 Marks [Int:15+Ext:60]**

Aims, Objectives and Outcomes

Linear programming problems are having wide applications in mathematics, statistics, computer science, economics, and in many social and managerial sciences. For mathematicians it is a sort of mathematical modelling process, for statisticians and economists it is useful for planning many economic activities such as transport of raw materials and finished products from one place to another with minimum cost and for military heads it is useful for scheduling the training activities and deployment of army personnel. The emphasis of this course is on nurturing the linear programming skills of students *via*. the algorithmic solution of small-scale problems, both in the general sense and in the specific applications where these problems naturally occur. On successful completion of this course, the students will be able to

- solve linear programming problems geometrically
- understand the drawbacks of geometric methods
- solve LP problems more effectively using Simplex algorithm *via*. the use of condensed tableau of A.W. Tucker
- convert certain related problems, not *directly* solvable by simplex method, into a form that can be attacked by simplex method.
- understand duality theory, a theory that establishes relationships between linear programming problems of maximization and minimization
- understand game theory
- solve transportation and assignment problems by algorithms that take advantage of the simpler nature of these problems

Syllabus

Text	Linear Programming and Its Applications: James K. Strayer *Undergraduate Texts in Mathematics Springer (1989) ISBN: 978-1-4612-6982-3*

Module-I (16 hrs)

Chapter1 Geometric Linear Programming: Profit Maximization and Cost Minimization, *typical motivating examples*, *mathematical formulation*, Canonical Forms for Linear Programming Problems, *objective functions, constraint set, feasible solution, optimal solution* , Polyhedral Convex Sets, *convex set, extreme point*, *theorems asserting existence of optimal solutions,* The Two Examples Revisited, *graphical solutions to the problems*, A Geometric Method for Linear Programming, *the difficulty in the method*, Concluding Remarks

Chapter2 The Simplex Algorithm:- Canonical Slack Forms for Linear Programming Problems; Tucker Tableaus, *slack variables, Tucker tableaus*, *independent variables* or *non basic variables, dependent variables* or *basic variables,* .An Example: Profit Maximization, *method of solving a typical canonical maximization problem*, The Pivot Transformation, *The Pivot Transformation for Maximum and Minimum Tableaus,* An Example: Cost Minimization, *method of solving a typical canonical minimization problem*, The Simplex Algorithm for Maximum Basic Feasible Tableaus, The Simplex Algorithm for Maximum Tableaus, Negative Transposition; The Simplex Algorithm for Minimum Tableaus, Cycling, Simplex Algorithm Anti cycling Rules, Concluding Remarks

Module-II (14 hrs)

Chapter3 Noncanonical Linear Programming Problems:- Unconstrained Variables, Equations of Constraint, Concluding Remarks

Chapter 4 : Duality Theory :- Duality in Canonical Tableaus, The Dual Simplex Algorithm, *The Dual Simplex Algorithm for Minimum Tableaus, The Dual Simplex Algorithm for Maximum Tableaus,* Matrix Formulation of Canonical Tableaus ,The Duality Equation, Duality in Noncanonical Tableaus, Concluding Remarks

Module-III (18 hrs)

Chapter 5 Matrix Games:- An Example; Two-Person Zero-Sum Matrix Games, Domination in a Matrix Game, Linear Programming Formulation of Matrix Games, The Von Neumann Minimax Theorem, The Example Revisited, Two More Examples, Concluding Remarks

Chapter 6 Transportation and Assignment Problems :- The Balanced Transportation Problem, The Vogel Advanced-Start Method (VAM), The Transportation Algorithm, Another Example, Unbalanced Transportation Problems, The Assignment Problem, *The Hungarian Algorithm,* Concluding Remarks, *The Minimum-Entry Method*, *The Northwest-Corner Method*

References:

1	RobertJ.Vanderbei:LinearProgramming:Foundations and Extensions (2/e) Springer Science+Business Media LLC(2001) ISBN: 978-1-4757-5664-7
2	Frederick S Hiller, Gerald J Lieberman: Introduction to Operation Research(10/e) *McGraw-Hill Education, 2 Penn Plaza, New York(2015)ISBN: 978-0-07-352345-3*
3	Paul R. Thie, G. E. Keough : An Introduction to Linear Programming and Game Theory(3/e) *John Wiley and Sons,Ins.(2008)ISBN: 978-0-470-23286-6*
4	Louis Brickman: Mathematical Introduction to Linear Programming and Game Theory *UTM,Springer Verlag,NY(1989)ISBN:0-387-96931-4*
5	Jiri Matoušek, Bernd Gartner: Understanding and Using Linear Programming *Universitext, Springer-Verlag Berlin Heidelberg (2007)ISBN: 978-3-540-30697-9*

MTS5 B09 INTRODUCTION TO GEOMETRY AND THEORY OF EQUATIONS

3 hours/week	3 Credits	75 Marks Int:15+Ext:60]

Aims, Objectives and Outcomes	

Geometry

Geometry is, basically, the study concerned with questions of shape, size, and relative position of planar and spatial objects. The classical Greek geometry, also known as *Euclidean geometry* after the work of Euclid, was once regarded as one of the highest points of rational thought, contributing to the thinking skills of logic, deductive reasoning and skills in problem solving.

In the early 17 th century, the works of Rene Descartes and Pierre de Fermat put the foundation stones for the creation of *analytic geometry* where the idea of a coordinate system was introduced to simplify the treatment of geometry and to solve a wide variety of geometric problems.

Desargues, a contemporary of Descartes was fascinated towards the efforts of artists/painters to give a realistic view of their art works/paintings usually done on a flat surface such as canvas or paper. To get a realistic view of a three dimensional object/scene depicted on a flat surface, a right impression of height, width, depth and position in relation to each other of the objects in the scene is required. This idea is called *perspective* in art. If two artists make perspective drawings of the same object, their drawings shall not be identical but there shall be certain properties of these drawings that remain the same or that remain *invariant*. The study of such invariant things crystallised into what is now called *projective geometry*. Now days, it plays a major role in computer graphics and in the design of camera models.

Another development is the evolution of *affine geometry*. In simple terms, if we look at the shadows of a rectangular window on the floor under sunlight, we could see the shadows not in perfect rectangular form but often in the shape of a parallelogram. The size of shadows also changes with respect to the position of the sun. Hence, neither length nor angle is *invariant* in the *transformation* process. However, the opposite sides of the images are always parallel. So this transformation keeps *parallelism* intact. The investigation of *invariants* of *all* shadows is the basic problem of affine geometry.

Towards the end of nineteenth century, there were several different geometries: Euclidean, affine, projective, inversive, spherical, hyperbolic, and elliptic to name a few. It was the idea of Felix Klein to bring the study of all these different geometries into a single platform. He viewed each geometry as a space together with a *group of transformations* of that space and regarded those properties of figures left unaltered by the group as geometrical properties. In this course, it is intended to take up a study of a few geometries based on the *philosophy* of Klein.

Theory of equations

Theory of equations is an important part of traditional algebra course and it mainly deals with polynomial equations and methods of finding their *algebraic solution* or *solution by radicals*. This means we seek a formula for solutions of polynomial equations in terms of coefficients of polynomials that involves only the operations of addition, subtraction, multiplication, division and taking roots. A well knitted formula for the solution of a quadratic polynomial equation is known to us from high school classes and is not difficult to derive. However, there is an increasing difficulty to derive such a formula for polynomial equations of third and fourth degree. One of our tasks in this learning process is to derive formulae for the solutions of *third* and *fourth* degree polynomial equations given by Carden and Ferrari respectively. In the mean time, we shall find out the relationship between the roots and coefficients of an n^{th} degree polynomial and an upper and lower limit for the roots of such a polynomial. This often help us to locate the region of solutions for a general polynomial equation. Methods to find out integral and rational roots of a general n^{th} degree polynomial with rational coefficients are also devised. However, all efforts to find out an *algebraic solution* for general polynomial equations of degree higher than fourth failed or didn't work. This was not because one failed to hit upon the right idea, but rather due to the disturbing fact that there was no such formula.

Upon successful completion of the course, students will be able to

- Understand several basic facts about parabola, hyperbola and ellipse *(conics)* such as their equation in standard form, focal length properties, and reflection properties, their tangents and normal.

- Recognise and classify conics.

- Understand Kleinian view of Euclidean geometry.

- Understand affine transformations, the inherent group structure, the idea of parallel projections and the basic properties of parallel projections.

- Understand the fundamental theorem of affine geometry.

- Learn to solve polynomial equations upto degree four.

Syllabus

Text (1) Geometry(2/e): David A Brannan, Mathew F Espen, Jeremy J Gray Cambridge University Press(2012) ISBN: 978-1-107-64783-1

Text (2) Theory of Equations : J V Uspensky McGraw Hill Book Company, Inc. (1948) ISBN:07-066735-7

Module- I	Text (1)	(20 hrs)

Conics

1.1.1: Conic Sections

1.1.3: Focus-Directrix Definition of the Non-Degenerate Conics- *definition*, *parabola in standard form, ellipse in standard form, hyperbola in standard form,* Rectangular Hyperbola, Polar Equation of a Conic

1.1.4: Focal Distance Properties of Ellipse and Hyperbola-*Sum of Focal Distances of Ellipse, Difference of Focal Distances of Hyperbola,*

1.2: Properties of Conics- Tangents, *equation of tangents to ellipse, hyperbola, and parabola, polar of a point w.r.t. unit circle, normal,* Reflections, *The Reflection Law*, Reflection Property of the Ellipse, Reflection Property of the Hyperbola, Reflection Property of the Parabola, Conics as envelopes of tangent families

1.3: Recognizing Conics- *equation of conic in general form, identifying a conic*

Affine Geometry

2.1: Geometry and Transformations - What is Euclidean Geometry? *Isometry, Euclidean properties, Euclidean transformation*, Euclidean-Congruence

2.2: Affine Transformations and Parallel Projections- Affine Transformations, *Basic Properties of Affine Transformations*, Parallel Projections, *Basic Properties of Parallel Projections*, Affine Geometry, *Midpoint Theorem, Conjugate Diameters Theorem, Affine Transformations and Parallel Projections, affine transformations as composite of two parallel projections*

2.3: Properties of Affine Transformations-Images of Sets Under Affine Transformations, The Fundamental Theorem of Affine Geometry (without proof), Proofs of the Basic Properties of Affine Transformations.

Module- II	Text (2)	(14 hrs)

Theory of Equations

Chapter II

II.3: Division of polynomials, *quotient and remainder*, *method of detached coefficients*

II.4: The remainder theorem

II.5: Synthetic Division

II.7: Taylor formula, *expansion of a polynomial in powers of* $x - c$

Chapter III

III.1: Algebraic equations, roots, maximum number of roots

III.2: Identity theorem

III. 3: The Fundamental theorem of Algebra (*statement only*), *factorisation to linear factors, multiplicity of roots*

III. 4: Imaginary roots of equations with real coefficients

III. 5: Relations between roots and coefficients

Chapter IV

IV.1: Limits of roots

IV.2: Method to find upper limit of positive roots

IV.3: Limit for moduli of roots [*only the method to find out upper limit from the auxiliary equation is required*; *derivation omitted*]

IV.4: Integral roots

IV.5: Rational roots

Module- III	Text (2)	(14 hrs)

Chapter V

V.1: What is the solution of an equation, *algebraic solution or solution by radical*

V.2: Carden's formula

V.3: Discussion of solution

V.4: Irreducible case

V.6: Solutions of biquadratic equations, *Ferrari method [example2 omitted]*

Chapter VI

VI.1: Object of the Chapter

VI.2: The sign of a polynomial for small and large values of variables- *locating roots of polynomial between two numbers having values of opposite sign- geometric illustration only-[rigorous reasoning in the starred section omitted]*

VI.4: Corollaries- *roots of odd and even degree polynomial, number of roots in an interval counted according to their multiplicity*

VI.5: Examples

VI.6: An important identity and lemma *[derivation not needed]*

VI.7: Rolle's theorem *[proof omitted]*, *use in separating roots*

VI.10: Descarte's rule of signs-*only statement and illustrations are required*

References:	
1	*George A Jennings: Modern Geometry with Applications Universitext, Springer (1994) ISBN:0-387-94222-X*
2	*Walter Meyer: Geometry and its Application(2/e)Elsever, Academic Press(2006) ISBN:0-12-369427-0*
3	*Judith N Cederberg : A Course in Modern Geometries(2/e) UTM,Springer (2001) ISBN: 978-1-4419-3193-1*
4	*Patric J Ryan: Euclidean and Non Euclidean Geometry-An Analytic ApproachCambridge University Press, International Student Edition (2009) ISBN:978-0-521-12707-3*
5	David C Kay: College Geometry: A Unified Approach CRC Press Tayloe and Francic Group(2011) ISBN: 978-1-4398-1912-8 (Ebook-PDF)
6	James R Smart: Modern Geometries(5/e) Brooks/Cole Publishing Co.,(1998) ISBN:0-534-35188-3
7	Michele Audin: Geometry Universitext, Springer(2003)ISBN:3-540-43498-4
8	Dickson L.E: Elementary Theory of Equations John Wiley and Sons,Inc. NY(1914)
9	Turnbull H.W: Theory of Equations(4/e) Oliver and Boyd Ltd Edinburg (1947)
10	Todhunter I: An Elementary Treatise on the Theory of Equations(3/e) Macmillan and Co. London(1875)
11	William Snow Burnside and Arthur William Panton: The Theory of Equations with an Introduction to Binary Algebraic Forms Dublin University Press Series (1881)

SIXTH SEMESTER

MTS6 B10 REAL ANALYSIS

5 hours/week	5 Credits	100 Marks [Int:20+Ext:80]

Aims, Objectives and Outcomes

The course is built upon the foundation laid in Basic Analysis course of fifth semester. The course thoroughly exposes one to the rigour and methods of an analysis course. One has to understand definitions and theorems of text and study examples well to acquire skills in various problem solving techniques. The course will teach one how to combine different definitions, theorems and techniques to solve problems one has never seen before. One shall acquire ability to realise when and how to apply a particular theorem and how to avoid common errors and pitfalls. The course will prepare students to formulate and present the ideas of mathematics and to communicate them elegantly.

On successful completion of the course, students will be able to

- State the definition of continuous functions, formulate sequential criteria for continuity and prove or disprove continuity of functions using this criteria.
- Understand several deep and fundamental results of continuous functions on intervals such as boundedness theorem, maximum-minimum theorem, intermediate value theorem, preservation of interval theorem and so on.
- Realise the difference between continuity and uniform continuity and equivalence of these ideas for functions on closed and bounded interval.
- Understand the significance of uniform continuity in continuous extension theorem.
- Develop the notion of Riemann integrability of a function using the idea of tagged partitions and calculate the integral value of some simple functions using the definition.
- Understand a few basic and fundamental results of integration theory.
- Formulate Cauchy criteria for integrability and a few applications of it. In particular they learn to use Cauchy criteria in proving the non integrability of certain functions.
- Understand classes of functions that are always integrable
- Understand two forms of fundamental theorem of calculus and their significance in the practical problem of evaluation of an integral.
- Find a justification for 'change of variable formula' used in the practical problem of evaluation of an integral.
- Prove convergence and divergence of sequences of functions and series

- Understand the difference between pointwise and uniform convergence of sequences and series of functions
- Answer a few questions related to interchange of limits.
- Learn and find out examples/counter examples to prove or disprove the validity of several mathematical statements that arise naturally in the process/context of learning.
- Understand the notion of improper integrals, their convergence, principal value and evaluation.
- Learn the properties of and relationship among two important improper integrals namely *beta and gamma functions* that frequently appear in mathematics, statistics, science and engineering.

Syllabus

Text(1)	Introduction to Real Analysis(4/e) : Robert G Bartle, Donald R Sherbert *John Wiley & Sons(2011) ISBN 978-0-471-43331-6*
Text(2)	Improper Riemann Integrals: Ioannis M. Roussos *CRC Press by Taylor & Francis Group, LLC(2014) ISBN: 978-1-4665-8808-0 (eBook - PDF)*

Module-I Text(1) (18 hrs)

5.1: Continuous Functions- *definition, sequential criteria for continuity, discontinuity criteria, examples of continuous and discontinuous functions, Dirichlet and Thomae function*

5.3: Continuous Functions on Intervals- Boundedness Theorem, The Maximum-Minimum Theorem, Location of Roots Theorem, Bolzano's Intermediate Value Theorem, Preservation of Intervals Theorem

5.4: Uniform Continuity- *definition, illustration*, Nonuniform Continuity Criteria, Uniform Continuity Theorem, Lipschitz Functions, *Uniform Continuity of Lipschitz Functions, converse,* The Continuous Extension Theorem, Approximation *by step functions & piecewise linear functions*, Weierstrass Approximation Theorem *(only statement)*

Module-II Text(1) (22 hrs)

7.1: Riemann Integral –Partitions and Tagged Partitions, *Riemann sum, Riemann integrability, examples,* Some Properties of the Integral, Boundedness Theorem

7.2: Riemann Integrable Functions-*Cauchy Criterion, illustrations*, The Squeeze Theorem, Classes of Riemann Integrable Functions, *integrability of continuous and monotone functions*, The Additivity Theorem

7.3: The Fundamental Theorem-The Fundamental Theorem (First Form), The Fundamental Theorem (Second Form), , Substitution Theorem, Lebesgue's Integrability Criterion, *Composition Theorem, The Product Theorem*, Integration by Parts, *Taylor's Theorem with the Remainder*

Module-III Text(1) (17 hrs)

8.1: Pointwise and Uniform Convergence-*definition, illustrations,* The Uniform Norm, *Cauchy Criterion for Uniform Convergence*

8.2: Interchange of Limits- *examples leading to the idea*, Interchange of Limit and Continuity, Interchange of Limit and Derivative *[only statement of theorem 8.2.3 required*; *proof omitted]* Interchange of Limit and Integral , Bounded convergence theorem(statement only) *[8.2.6 Dini's theorem omitted]*

9.4: Series of Functions – *(A quick review of series of real numbers of section 3.7 without proof) definition, sequence of partial sum, convergence, absolute and uniform convergence*, Tests for Uniform Convergence , Weierstrass M-Test *(only upto and including 9.4.6)*

Module-IV Text(2) (23 hrs)

Improper Riemann Integrals

1.1: Definitions and Examples

1.2: Cauchy Principal Value

1.3: Some Criteria of Existence

2.1: Calculus Techniques ['2.1.1 Applications' Omitted]

2.2: Integrals Dependent on Parameters- *upto and including example 2.2.4*

2.6: The Real Gamma and Beta Functions- *upto and including Example 2.6.18*

References:

1	Charles G. Denlinger: Elements of Real Analysis *Jones and Bartlett Publishers Sudbury, Massachusetts (2011) ISBN:0-7637-7947-4 [Indian edition: ISBN- 9380853157]*
2	David Alexander Brannan: A First Course in Mathematical Analysis *Cambridge University Press,US(2006) ISBN: 9780521684248*
3	John M. Howie: Real Analysis *Springer Science & Business Media(2012)[Springer Undergraduate Mathematics Series] ISBN: 1447103416*
4	James S. Howland: Basic Real Analysis *Jones and Bartlett Publishers Sudbury, Massachusetts (2010) ISBN:0-7637-7318-2*
5	Terence Tao: Analysis I & II (3/e) *TRIM 37 & 38 Springer Science+Business Media Singapore 2016; Hindustan book agency(2015) ISBN 978-981-10-1789-6 (eBook) & ISBN 978-981-10-1804-6 (eBook)*
6	Richard R Goldberg: Methods of Real Analysis *Oxford and IBH Publishing Co.Pvt.Ltd. NewDelhi(1970)*
7	Saminathan Ponnusamy: Foundations of Mathematical Analysis *Birkhauser(2012) ISBN 978-0-8176-8291-0*
8	William F Trench: Introduction to Real Analysis *ISBN 0-13-045786-8*
9	Ajith Kumar & S Kumaresan : A Basic Course in Real Analysis *CRC Press, Taylor & Francis Group(2014) ISBN: 978-1-4822-1638-7 (eBook - PDF)*
10	Hugo D Junghenn : A Course in Real Analysis *CRC Press, Taylor & Francis Group(2015) ISBN: 978-1-4822-1928-9 (eBook - PDF)*

MTS6 B11 COMPLEX ANALYSIS

5 hours/week	5 Credits	100 Marks [Int:20+Ext:80]

Aims, Objectives and Outcomes	

The course is aimed to provide a thorough understanding of complex function theory. It is intended to develop the topics in a fashion analogous to the calculus of real functions. At the same time differences in both theories are clearly emphasised. When real numbers are replaced by complex numbers in the definition of derivative of a function, the resulting *complex differentiable functions* (more precisely *analytic functions)* turn out to have many remarkable properties not possessed by their real analogues. These functions have numerous applications in several areas of mathematics such as differential equations, number theory etc. and also in science and engineering. The focus of the course is on the study of analytic functions and their basic behaviour with respect to the theory of complex calculus.

The course enables students

- to understand the difference between differentiability and analyticity of a complex function and construct examples.
- to understand necessary and sufficient condition for checking analyticity.
- to know of harmonic functions and their connection with analytic functions
- to know a few elementary analytic functions of complex analysis and their properties.
- to understand definition of complex integral, its properties and evaluation.
- to know a few fundamental results on contour integration theory such as Cauchy's theorem,
  Cauchy-Goursat theorem and their applications.
- to understand and apply Cauchy's integral formula and a few consequences of it such as Liouville's theorem, Morera's theorem and so forth in various situations.
- to see the application of Cauchy's integral formula in the derivation of power series expansion of an analytic function.
- to know a more general type of series expansion analogous to power series expansion *viz.*
  Laurent's series expansion for functions having *singularity*.
- to understand how Laurent's series expansion lead to the concept of *residue,* which in turn provide another fruitful way to evaluate complex integrals and, in some cases, even real integrals.
- to see another application of residue theory in locating the region of zeros of an analytic function.

Syllabus

Text Complex Analysis A First Course with Applications (3/e): Dennis Zill & Patric Shanahan *Jones and Bartlett Learning(2015)ISBN:1- 4496-9461-6*

Module- I (25 hrs)

Analytic Functions

3.1: Limit and Continuity- *Limit of a complex function*, *condition for non existence of limit, real and imaginary parts of limit, properties of complex limits, continuity, discontinuity of principal square root function,* properties of continuous functions, *continuity of polynomial and rational functions*, Bounded Functions, Branches, Branch Cuts and Points

3.2: Differentiability and Analyticity – *Derivative of a complex Function, rules of differentiation, function that is nowhere differentiable,* Analytic functions, entire functions, singular points, *Analyticity of sum product and quotient,* L'Hospital rule

3.3: Cauchy Riemann Equations- Necessary condition for analyticity, *Criterion for non analyticity*, sufficient condition for analyticity, *sufficient condition for differentiability, Cauchy Riemann equations in polar coordinates*

3.4: Harmonic Functions- *definition, analyticity and harmonic nature,* harmonic conjugate functions, *finding harmonic conjugate*

Elementary Functions

4.1: Exponential and logarithmic functions-Complex Exponential Function, *its derivative*, *analyticity*, *modulus argument and conjugate*, *algebraic properties*, *periodicity*, exponential mapping *and its properties,* Complex Logarithmic Function, *logarithmic identities*, principal value of a complex logarithm, *Ln z* as inverse function, derivative, logarithmic mapping, *properties, other branches*

4.3: Trigonometric and Hyperbolic functions- Complex Trigonometric Functions, identities, *periodicity of sine and cosine*, Trigonometric equations *and their solution,* Modulus, zeroes analyticity, *[subsection 'Trigonometric Mapping' omitted]*, Complex Hyperbolic Functions, relation to *sine* and *cosine*

Module- II (20 hrs)

Integration in the Complex plane

5.1: Real Integrals- Definite Integral, *simple, smooth, closed curves*, Line integrals in the plane, Method of Evaluation-*curves defined parametrically and curves given as functions*, Orientation of a Curve

5.2: Complex Integral-contours, *definition of* complex integral, complex valued function of a real variable, evaluation of contour integral, properties *of contour integral, ML-inequality*

5.3: Cauchy-Goursat Theorem- simply and multiply connected regions, Cauchy theorem, Cauchy-Goursat theorem *for simply connected domain (without proof)*, Multiply Connected Domains, *principe of deformation of contours*, *Cauchy-Goursat theorem for multiply connected domains, illustrations*

5.4: Independence of Path- *definition, analyticity and path independence, anti derivative*, *Fundamental theorem for contour integrals*, Some Conclusions, *Existence of anti derivative*

5.5: Cauchy's Integral Formulas & their Consequences- Cauchy's Two Integral Formulas, *illustration of their use*, Some Consequences of the Integral Formulas-cauchy's inequality, Liouville theorem, Morera's theorem, Maximum modulus theorem

Module- III (15 hrs)

Series

6.1: Sequences and Series- *definition, criteria for convergence*, Geometric series, *necessary condition for convergence*, test for divergence, *absolute and conditional convergence, Ratio test, root test*, Power Series, *circle of convergence, radius of convergence*, Arithmetic of Power Series

6.2: Taylor Series- differentiation and integration of power series, *term by term differentiation and integration*, Taylor Seties, *Maclaurian series* , illustrations

6.3: Laurent's Series- isolated singularities, *Laurent's Theorem [proof omitted], illustrations*

Module- IV (20 hrs)

Residues

6.4: Zeros and Poles- classification of isolated singular points, *removable singularity, pole, essential singularity, order of zeros and poles*

6.5: Residues and Residue Theorem- residue , *method of evaluation of residue at poles,* (Cauchy's) Residue Theorem, *illustrations*

6.6: Some Consequences of Residue theorem-

6.6.1:Evaluation of Real Trigonometric Integrals

References:	
1	*James Ward Brown, Ruel Vance Churchill: Complex variables and applications(8/e) McGraw-Hill Higher Education, (2009) ISBN: 0073051942*
2	*Alan Jeffrey: Complex Analysis and Applications(2/e) Chapman and Hall/CRC Taylor Francis Group(2006)ISBN:978-1-58488-553-5*
3	*Saminathan Ponnusamy, Herb Silverman: Complex Variables with Applications Birkhauser Boston(2006) ISBN:0-8176-4457-4*
4	*John H. Mathews & Russell W. Howell : Complex Analysis for Mathematics and Engineering (6 /e)*
5	H A Priestly : Introduction to Complex Analysis(2/e) *Oxford University Press(2003)ISBN: 0 19 852562 1*
6	Jerrold E Marsden, Michael J Hoffman: Basic Complex Analysis(3/e) *W.H Freeman,N.Y.(1999) ISBN:0-7167- 2877- X*

SIXTH SEMESTER

MTS6 B12 CALCULUS OF MULTI VARIABLE

5 hours/week	4 Credits	100 Marks [Int:20+Ext:80]

Aims, Objectives and Outcomes

The intention of the course is to extend the immensely useful ideas and notions such as limit, continuity, derivative and integral seen in the context of function of single variable to function of several variables. The corresponding results will be the higher dimensional analogues of what we learned in the case of single variable functions. The results we develop in the course of calculus of multivariable is extremely useful in several areas of science and technology as many functions that arise in real life situations are functions of multivariable.

The successful completion of the course will enable the student to

- Understand several contexts of appearance of multivariable functions and their representation using graph and contour diagrams.
- Formulate and work on the idea of limit and continuity for functions of several variables.
- Understand the notion of *partial derivative*, their computation and interpretation.
- Understand chain rule for calculating partial derivatives.
- Get the idea of *directional derivative,* its evaluation, interpretation, *and* relationship with partial derivatives.
- Understand the concept of *gradient*, a few of its properties, application and interpretation.
- Understand the use of partial derivatives in getting information of tangent plane and normal line.
- Calculate the maximum and minimum values of a multivariable function using second derivative test and Lagrange multiplier method.
- Find a few real life applications of Lagrange multiplier method in optimization problems.
- Extend the notion of integral of a function of single variable to integral of functions of two and three variables.
- Address the practical problem of evaluation of double and triple integral using Fubini's theorem and change of variable formula.
- Realise the advantage of choosing other coordinate systems such as polar, spherical, cylindrical etc. in the evaluation of double and triple integrals .
- See a few applications of double and triple integral in the problem of finding out surface area ,mass of lamina, volume, centre of mass and so on.
- Understand the notion of a vector field, the idea of curl and divergence of a vector field, their evaluation and interpretation.
- Understand the idea of line integral and surface integral and their evaluations.
- Learn three major results viz. Green's theorem, Gauss's theorem and Stokes' theorem of multivariable calculus and their use in several areas and directions.

Syllabus

Text	Calculus: Soo T Tan *Brooks/Cole, Cengage Learning (2010) ISBN 0-534-46579-X)*

Module-I (18 hrs)

13.1: Functions of two or more variables- Functions of Two Variables, Graphs of Functions of Two Variables, Level Curves, Functions of Three Variables and Level Surfaces

13.2: Limits and continuity-An Intuitive Definition of a Limit, *existence and non existence of limit,* Continuity of a Function of Two Variables, Continuity on a Set, *continuity of polynomial and rational functions*, *continuity of composite functions,* Functions of Three or More Variables, The $\varepsilon - \delta$ Definition of a Limit

13.3: Partial Derivatives- Partial Derivatives of Functions of Two Variables, *geometric interpretation*, Computing Partial Derivatives, Implicit Differentiation, Partial Derivatives of Functions of More Than Two Variables, Higher-Order Derivatives, *clairaut theorem, harmonic functions*

13.4: Differentials- Increments, The Total Differential, *interpretation,* Error in Approximating Δz by dz *[only statement of theorem1 required ; proof omitted]* Differentiability of a Function of Two Variables, *criteria*, Differentiability and Continuity, Functions of Three or More Variables

13.5: The Chain rule- The Chain Rule for Functions Involving One Independent Variable, The Chain Rule for Functions Involving Two Independent Variables, The General Chain Rule, Implicit Differentiation

Module-II (16 hrs)

13.6: Directional Derivatives and Gradient vectors - The Directional Derivative, The Gradient of a Function of Two Variables, Properties of the Gradient, Functions of Three Variables

13.7: Tangent Planes and Normal Lines- Geometric Interpretation of the Gradient, Tangent Planes and Normal Lines, Using the Tangent Plane of f to approximate the Surface $z = f(x, y)$

13.8: Extrema of Functions of two variables - Relative and Absolute Extrema, Critical Points—Candidates for Relative Extrema, The Second

Derivative Test for Relative Extrema, Finding the Absolute Extremum Values of a Continuous Function on a Closed Set

13.9: Lagrange Multipliers- Constrained Maxima and Minima, The Method of Lagrange Multipliers, *Lagrange theorem,* Optimizing a Function Subject to Two Constraints

Module-III	**(21 hrs)**

14.1: Double integrals- An Introductory Example, Volume of a Solid Between a Surface and a Rectangle, The Double Integral Over a Rectangular Region, Double Integrals Over General Regions, Properties of Double Integrals

14.2: Iterated Integrals-Iterated Integrals Over Rectangular Regions, Fubini's Theorem for Rectangular Regions, Iterated Integrals Over Nonrectangular Regions, *y- simple and x- simple regions, advantage of changing the order of integration*

14.3:Double integrals in polar coordinates- Polar Rectangles, Double Integrals Over Polar Rectangles, Double Integrals Over General Regions, r-simple region, method of evaluation

14.4: Applications of Double integral- Mass of a Lamina, Moments and Center of Mass of a Lamina, Moments of Inertia, Radius of Gyration of a Lamina

14.5: Surface Area- Area of a Surface $z = f(x, y)$, Area of Surfaces with Equations $y = g(x, z)$ and $x = h(y, z)$

14.6: Triple integrals- Triple Integrals Over a Rectangular Box, *definition, method of evaluation as iterated integrals,* Triple Integrals Over General Bounded Regions in Space, Evaluating Triple Integrals Over General Regions, *evaluation technique,* Volume, Mass, Center of Mass, and Moments of Inertia

14.7: Triple Integrals in cylindrical and spherical coordinates- *evaluation of integrals in* Cylindrical Coordinates, Spherical Coordinates

14.8: Change of variables in multiple integrals- Transformations, Change of Variables in Double Integrals *[only the method is required; derivation omitted], illustrations*, Change of Variables in Triple Integrals

Module-IV (25 hrs)

15.1: Vector Fields- *V.F. in two and three dimensional space,* Conservative Vector Fields

15.2: Divergence and Curl- *Divergence- idea and definition*, Curl- *idea and definition*

15.3: Line Integrals- *Line integral w.r.t. arc length-motivation, basic idea and definition*, Line Integrals with Respect to Coordinate Variables, *orientation of curve* Line Integrals in Space, Line Integrals of Vector Fields

15.4: Independence of Path and Conservative Vector Fields-*path independence through example, definition, fundamental theorem for line integral,* Line Integrals Along Closed Paths, *work done by conservative vector field,* Independence of Path and Conservative Vector Fields, Determining Whether a Vector Field Is Conservative, *test for conservative vector field* Finding a Potential Function, Conservation of Energy

15.5: Green's Theorem- Green's Theorem for Simple Regions, *proof of theorem for simple regions, finding area using line integral,* Green's Theorem for More General Regions, Vector Form of Green's Theorem

15.6: Parametric Surfaces-Why We Use Parametric Surfaces, Finding Parametric Representations of Surfaces, Tangent Planes to Parametric Surfaces, Area of a Parametric Surface *[derivation of formula omitted]*

15.7: Surface Integrals-Surface Integrals of Scalar Fields, *evaluation of surface integral for surfaces that are graphs , [derivation of formula omitted; only method required]* Parametric Surfaces, *evaluation of surface integral for parametric surface*, Oriented Surfaces, Surface Integrals of Vector Fields- *definition, flux integral*, *evaluation of surface integral for graph [method only],* Parametric Surfaces, *evaluation of surface integral of a vector field for parametric surface [method only]*

15.8: The Divergence Theorem-*divergence theorem for simple solid regions (statement only)*, *illustrations*, Interpretation of Divergence

15.9: Stokes Theorem-*generalization of Green's theorem –Stokes Theorem*, *illustrations*, Interpretation of Curl

References:

1	JoelHass, Christopher Heil & Maurice D. Weir : Thomas' Calculus(14/e) *Pearson(2018) ISBN 0134438981*
2	Robert A Adams & Christopher Essex : Calculus: *A complete Course (8/e) Pearson Education Canada (2013) ISBN:* 032187742X
3	Jon Rogawski: Multivariable Calculus *Early Transcendentals* (2/e) W. H. Freeman and Company(2012) *ISBN: 1-4292-3187-4*
4	Anton, Bivens & Davis : Calculus *Early Transcendentals* (10/e) John Wiley & Sons, Inc.(2012) ISBN: 978-0-470-64769-1
5	James Stewart : Calculus (8/e) *Brooks/Cole Cengage Learning(2016) ISBN: 978-1-285-74062-1*
6	Jerrold E. Marsden & Anthony Tromba :Vector Calculus (6/e) *W. H. Freeman and Company ,New York(2012) ISBN: 978-1-4292-1508-4*
7	Arnold Ostebee & Paul Zorn: Multivariable Calculus (2/e) *W. H. Freeman Custom Publishing, N.Y.(2008)ISBN: 978-1-4292-3033-9*

SIXTH SEMESTER

MTS6 B13 DIFFERENTIAL EQUATIONS

5 hours/week	**4 Credits**	**100 Marks [Int:20+Ext:80]**

Aims, Objectives and Outcomes

Differential equations model the physical world around us. Many of the laws or principles governing natural phenomenon are statements or relations involving rate at which one quantity changes with respect to another. The mathematical formulation of such relations (*modelling*) often results in an equation involving derivative (*differential equations*). The course is intended to find out ways and means for solving differential equations and the topic has wide range of applications in physics, chemistry, biology, medicine, economics and engineering.

On successful completion of the course, the students shall acquire the following skills/knowledge.

- Students could identify a number of areas where the modelling process results in a differential equation.
- They will learn what an ODE is, what it means by its solution, how to classify DEs, what it means by an IVP and so on.
- They will learn to solve DEs that are in linear, separable and in exact forms and also to analyse the solution.
- They will realise the basic differences between linear and non linear DEs and also basic results that guarantees a solution in each case.
- They will learn a method to approximate the solution successively of a first order IVP.
- They will become familiar with the theory and method of solving a second order linear homogeneous and nonhomogeneous equation with constant coefficients.
- They will learn to find out a *series solution* for homogeneous equations with variable coefficients near *ordinary points.*
- Students acquire the knowledge of solving a differential equation using Laplace method which is especially suitable to deal with problems arising in engineering field.
- Students learn the technique of solving *partial differential equations* using the method of separation of variables

Syllabus

Text	Elementary Differential Equations and Boundary Value Problems (11/e): William E Boyce, Richard C Diprima And Douglas B Meade *John Wiley & Sons(2017) ISBN: 1119169879*

Module-I (22 hrs)

1.1: Some Basic Mathematical Models; Direction Fields

1.2: Solutions of some Differential equations

1.3: Classification of Differential Equations

2.1: Linear Differential Equations; Method of Integrating Factors

2.2: Separable Differential Equations

2.3: Modelling with First Order Differential Equations

2.4: Differences Between Linear and Nonlinear Differential Equations

2.6: Exact Differential Equations and Integrating Factors

2.8: The Existence and Uniqueness Theorem *(proof omitted)*

Module-II (23 hrs)

3.1: Homogeneous Differential Equations with Constant Coefficients

3.2: Solutions of Linear Homogeneous Equations; the Wronskian

3.3: Complex Roots of the Characteristic Equation

3.4: Repeated Roots; Reduction of Order

3.5: Nonhomogeneous Equations; Method of Undetermined Coefficients

3.6: Variation of Parameters

5.2: Series solution near an ordinary point, part1

5.3: Series solution near an ordinary point,part2

Module-III (15 hrs)

6.1: Definition of the Laplace Transform

6.2: Solution of Initial Value Problems

6.3: Step Functions

6.5: Impulse Functions

6.6: The Convolution Integral

Module-IV	**(20 hrs)**

10.1: Two-Point Boundary Value Problems

10.2: Fourier Series

10.3: The Fourier Convergence Theorem

10.4: Even and Odd Functions

10.5: Separation of Variables; Heat Conduction in a Rod

10.7: The Wave Equation: Vibrations of an Elastic String

References:

1	Dennis G Zill &Michael R Cullen: Differential Equations with Boundary Value Problems(7/e):*Brooks/Cole Cengage Learning(2009)ISBN: 0-495-10836-7*
2	R Kent Nagle, Edward B. Saff & Arthur David Snider: Fundamentals of Differential Equations(8/e) *Addison-Wesley(2012) ISBN: 0-321-74773-9*
3	C. Henry Edwards &David E. Penney: Elementary Differential Equations (6/e) *Pearson Education, Inc. New Jersey (2008) ISBN 0-13-239730-7*
4	John Polking, Albert Boggess & David Arnold : Differential Equations with Boundary Value Problems(2/e) *Pearson Education, Inc New Jersey(2006) ISBN 0-13-186236-7*
5	Henry J. Ricardo: A Modern Introduction to Differential Equations(2/e) *Elsevier Academic Press(2009)ISBN: 978-0-12-374746-4*
6	James C Robinson: An Introduction to Ordinary Differential Equations *Cambridge University Press (2004)ISBN: 0-521-53391-0*

SIXTH SEMESTER (Elective)

MTS6 B14 (E01) GRAPH THEORY

3 hours/week	**2 Credits**	**75 Marks [Int:15+Ext:60]**

Text	A First Look at Graph Theory: John Clark & Derek Allan Holton, *Allied Publishers, First Indian Reprint 1995*

Module-I (16 hrs)

1.1 Definition of a graph
1.2 Graphs as models
1.3 More definitions
1.4 Vertex degrees
1.5 Sub graphs
1.6 Paths and Cycles
1.7 Matrix representation of a graph *[up to Theorem 1.6 ; proof of Theorem 1.5 is omitted]*

Module-II (16 hrs)

2.1 Definitions and Simple Properties
2.2 Bridges *[Proof of Theorem 2.6 and Theorem 2.9 are omitted]*
2.3 Spanning Trees
2.6 Cut Vertices and Connectivity *[Proof of Theorem 2.21omitted]*

Module-III (16 hrs)

3.1 Euler Tour *[up to Theorem 3.2, proof of Theorem 3.2 omitted]*
3.3:Hamiltonian Graphs *[Proof of Theorem 3.6 omitted]*
5.1:Plane and Planar graphs *[Proof of Theorem 5.1 omitted]*
5.2 Euler's Formula *[Proofs of Theorems 5.3 and Theorem 5.6 omitted]*

References:

1	R.J. Wilson: Introduction to Graph Theory, 4th ed., *LPE, Pearson Education*
2	J.A. Bondy& U.S.R. Murty : Graph Theory with Applications
3	J. Clark & D.A. Holton: A First Look at Graph Theory, *Allied Publishers*
4	N. Deo : Graph Theory with Application to Engineering and Computer Science, *PHI.*

FIFTH SEMESTER (OPEN COURSE)
(For students not having Mathematics as Core Course)

MTS5 D03 LINEAR MATHEMATICAL MODELS

3 hours/week	3 credits	75marks [Int:15+Ext:60]

Text	Finite Mathematics and Calculus with Applications (9/e) *Margaret L. Lial, Raymond N. Greenwell & Nathan P. Ritchey* Pearson Education, Inc(2012) ISBN: 0-321-74908-1

Module I 18 hrs

Chapter-1 Linear Functions

1.1: Slopes and Equations of Lines

1.2: Linear Functions and Applications

1.3: The Least Squares Line

Chapter-2 Systems of Linear Equations and Matrices

2.1: Solution of Linear Systems by the Echelon Method

2.2: Solution of Linear Systems by the Gauss-Jordan Method

2.3: Addition and Subtraction of Matrices

2.4: Multiplication of Matrices

2.5: Matrix Inverses

2.6: Input-Output Models

Module II 12 hrs

Chapter-3 Linear Programming: The Graphical Method

3.1: Graphing Linear Inequalities

3.2: Solving Linear Programming Problems Graphically

3.3 : Applications of Linear Programming

Module III 18 hrs

Chapter-4 Linear Programming: The Simplex Method

4.1: Slack Variables and the Pivot

4.2: Maximization Problems

4.3: Minimization Problems; Duality

4.4 :Nonstandard Problems

References:	
1	Soo T Tan: Finite Mathematics For the Managerial, Life, and social sciences(11/e) *Cengage Learning(2015) ISBN: 1-285-46465-6*
2	Ronald J. Harshbarger, James J. Reynolds: Mathematical Applications for the Management, Life, and Social Sciences (9/e) *Brooks/Cole Cengage Learning(2009) ISBN: 978-0-547-14509-9*
3	Stefan Waner, Steven R. Costenoble: Finite Mathematics and Applied Calculus(5/e) *Brooks/Cole Cengage Learning(2011) ISBN: 978-1-4390-4925-9*
4	Seymour Lipschutz, John J. Schiller, R. Alu Srinivasan: Beginning Finite Mathematics *Schaum's Outline Series, McGraw-Hill(2005)*
5	Howard L. Rolf: Finite Mathematics *Enhanced Edition*(7/e) *Brooks/Cole, Cengage Learning(2011) ISBN:978-0-538-49732-9*
6	Michael Sullivan: Finite Mathematics An Applied Approach(11/e*) John Wiley & Sons, Inc(2011)ISBN: 978-0470-45827-3*

FIRST SEMESTER

MTS1 C01:MATHEMATICS-1

4 hours/week **3 Credits** **75 Marks[Int.15 + Ext. 60]**

Text (1)	Calculus I (2/e) : Jerrold Marsden & Alan Weinstein *Springer-Verlag New York Inc(1985) ISBN 0-387-90974-5*
Text (2)	Calculus II (2/e) : Jerrold Marsden & Alan Weinstein *Springer-Verlag New York Inc(1985) ISBN 0-387-90975-3*

Module I 14 hrs

1.1: Introduction to the derivative-instantaneous velocity, slope of tangent line, differentiating simplest functions

1.2: Limits- Notion of limit, basic properties, derived properties, continuity, continuity of rational functions, *one sided limit, limit involving* $\pm\infty$

1.3: The derivative as Limit- formal definition, *examples*, *differentiability and continuity*, Leibnitz notation,

1.4: Differentiating Polynomials-power rule, sum rule etc.,

1.5: Product and quotients- product, quotient, reciprocal & integral power rule

1.6: Linear Approximation and Tangent Lines- equation of tangent line and linear approximation, *illustrations*

Module II 13 hrs

2.1: Rate of change and Second derivative- linear or proportional change, rates of change, second derivative,

2.2: The Chain Rule- power of a function rule, chain rule,

2.3: Fractional Power & Implicit Differentiation-rational power of a function rule, implicit differentiation

2.4: Related rates and parametric curves- Related rates, parametric curves, *word problems involving related rates*

2.5: Anti derivatives- anti differentiation and indefinite integrals, anti differentiation rules

Module III 18 hrs

3.1: Continuity and Intermediate value theorem-IVT: first and second version

3.2: Increasing and decreasing function- Increasing and decreasing test, critical point test, first derivative test

3.3: Second derivative and concavity- second derivative test for local maxima , minima and concavity , inflection points

3.4: Drawing of Graphs- graphing procedure, *asymptotic behaviour*

3.5: Maximum- Minimum Problems- maximum and minimum values on intervals, extreme value theorem, closed interval test, *word problems*

3.6: The Mean Value Theorem- The MVT, consequences of MVT-*Rolles Theorem, horserace theorem*

11.2: L'Hospital rule- Preliminary version, strengthened version

Module IV 19 hrs

4.1: Summation- summation, *distance and velocity*, properties of summation, telescoping sum (quick introduction- relevant ideas only)

4.2: Sums and Areas-step functions, area under graph *and its counterpart in distance-velocity problem*

4.3: The definition of Integral- signed area *(The counterpart of signed area for our distance-velocity problem),* The integral, Riemann sums

4.4: The Fundamental Theorem of Calculus-*Arriving at FTC intuitively using distance velocity problem,* Fundamental integration Method, *proof of FTC*, Area under graph, displacements and velocity

4.5: Definite and Indefinite integral-indefinite integral test, properties of definite integral, fundamental theorem of calculus: alternative version *(interpretation and explanation in terms of areas)*

4.6: Applications of the Integral- Area between graphs, area between intersecting graphs, total changes from rates of change,

9.1: Volume by slice method- the slice method, volume of solid of revolution by Disk method

9.3: Average Values and the Mean Value Theorem for Integrals- *motivation and definition of average value, illustration, geometric and physical interpretation,* the Mean Value Theorem for Integrals

References:	
1	Soo T Tan: Calculus *Brooks/Cole, Cengage Learning(2010)ISBN 0-534-46579-X*
2	Gilbert Strang: Calculus *Wellesley Cambridge Press(1991)ISBN:0-9614088-2-0*
3	Ron Larson. Bruce Edwards: Calculus(11/e) *Cengage Learning(2018) ISBN: 978-1-337-27534-7*
4	Robert A Adams & Christopher Essex : Calculus *Single Variable (8/e) Pearson Education Canada (2013) ISBN:* 0321877403
5	Joel Hass, Christopher Heil & Maurice D. Weir : Thomas' Calculus(14/e) *Pearson (2018) ISBN 0134438981*
6	Jon Rogawski & Colin Adams : Calculus *Early Transcendentals* (3/e) *W. H. Freeman and Company(2015) ISBN: 1319116450*

SECOND SEMESTER

MTS2 C02:MATHEMATICS-2

4 hours/week	**3 Credits**	**75 Marks [Int:15+Ext:60]**

Syllabus

Text (1)	Calculus I (2/e) : Jerrold Marsden & Alan Weinstein *Springer-Verlag New York Inc(1985) ISBN 0-387-90974-5*
Text (2)	Calculus II (2/e) : Jerrold Marsden & Alan Weinstein *Springer-Verlag New York Inc(1985) ISBN 0-387-90975-3*
Text (3)	Advanced Engineering Mathematics(6/e) : Dennis G Zill *Jones & Bartlett Learning, LLC(2018)ISBN: 978-1-284-10590-2*

Module- I	**Text (1) & (2)**	**14 hrs**

5.1: Polar coordinates and Trigonometry – Cartesian and polar coordinates

(*Only representation of points in polar coordinates, relationship between Cartesian and polar coordinates, converting from one system to another and regions represented by inequalities in polar system are required*)

5.3 : Inverse functions-inverse function test, inverse function rule

5.6: Graphing in polar coordinates- *Checking symmetry of graphs given in polar equation, drawings*, tangents to graph in polar coordinates

8.3: Hyperbolic functions- hyperbolic sine, cosine, tan etc., derivatives, anti differentiation formulas

8.4: Inverse hyperbolic functions- inverse hyperbolic functions *(their derivatives and anti derivatives)*

10.3: Arc length and surface area- Length of curves, Area of surface of revolution about x and y axes

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Module- II	**Text (2)**	**17 hrs**

11.3: Improper integrals- integrals over unbounded intervals, comparison test, integrals of unbounded functions

11.4: Limit of sequences and Newton's method ε-N definition, limit of powers, comparison test, Newton's method

11.5: Numerical Integration- Riemann Sum, Trapezoidal Rule, Simpson's Rule

12.1: The sum of an infinite series- convergence of series, properties of limit of sequences (*statements only*), geometric series, algebraic rules for series, the i^{th} term test

12.2: The comparison test and alternating series- comparison test, ratio comparison test, alternating series, alternating series test, absolute and conditional convergence

Module- III	**Text (3)**	**19 hrs**

7.6: Vector spaces – *definition, examples , subspaces, basis, dimension, span*

7.7: Gram-Schmidt Orthogonalization Process- *orthonormal bases for* $\mathbb{R}^n$, construction of orthonomal basis of $\mathbb{R}^n$

8.2: Systems of Linear Algebraic Equations- General form, solving systems, augmented matrix, Elementary row operations, Elimination Methods- *Gaussian elimination, Gauss–Jordan elimination, row echelon form, reduced row echelon form, inconsistent system*, networks, homogeneous system, *over and underdetermined system*

8.3: Rank of a Matrix- *definition*, row space, rank by row reduction, rank and linear system, *consistency of linear system*

8.4: Determinants- *definition*, *cofactor* *(quick introduction)*

8.5: Properties of determinant- properties, *evaluation of determinant by* row reducing to triangular form

Module- IV	**Text (3)**	**14 hrs**

8.6: Inverse of a Matrix – finding inverse, properties *of inverse*, adjoint method, row operations method, using inverse to solve a linear system

8.8: The eigenvalue problem- Definition, *finding eigenvalues and eigenvectors*, complex eigenvalues, eigenvalues and singular matrices, eigenvalues of inverse

8.9: Powers of Matrices- *Cayley Hamilton theorem*, finding the inverse

8.10: Orthogonal Matrices- symmetric matrices and eigenvalues, inner product, *criterion for* orthogonal matrix, construction of orthogonal matrix

8.12: Diagonalization- diagonalizable matrix-*sufficient conditions*, orthogonal diagonalizability *of symmetric matrix*, Quadratic Forms

References:	
1	*Soo T Tan: Calculus Brooks/Cole, Cengage Learning(2010)ISBN 0-534-46579-X*
2	*Gilbert Strang: Calculus Wellesley Cambridge Press(1991)ISBN:0-9614088-2-0*
3	*Ron Larson. Bruce Edwards: Calculus(11/e) Cengage Learning(2018) ISBN: 978-1-337-27534-7*
4	*Robert A Adams & Christopher Essex : Calculus Single Variable (8/e) Pearson Education Canada (2013) ISBN: 0321877403*
5	*Joel Hass, Christopher Heil & Maurice D. Weir : Thomas' Calculus(14/e) Pearson (2018) ISBN 0134438981*
6	*Advanced Engineering Mathematics(7/e) Peter V O'Neil: Cengage Learning(2012)ISBN: 978-1-111-42741-2*
7	Erwin Kreyszig: Advanced Engineering Mathematics(10/e) *John Wiley & Sons(2011) ISBN: 978-0-470-45836-5*
8	Glyn James: Advanced Modern Engineering Mathematics(4/e) *Pearson Education Limited(2011) ISBN: 978-0-273-71923-6*

THIRD SEMESTER

MTS3 C03:MATHEMATICS-3

5 hours/week	3 Credits	75 Marks[Int.15 + Ext. 60]

Text	Advanced Engineering Mathematics(6/e) : Dennis G Zill *Jones & Bartlett Learning, LLC(2018)ISBN: 978-1-284-10590-2*

Module I 21 hrs

9.1: Vector Functions – Vector-Valued Functions, Limits, Continuity, and Derivatives, Geometric Interpretation of $r'(t)$, Higher-Order Derivatives, Integrals of Vector Functions, Length of a Space Curve, Arc Length as a Parameter

9.2: Motion on a Curve-Velocity and Acceleration, Centripetal Acceleration, Curvilinear Motion in the Plane

9.3: Curvature and components of Acceleration- *definition, Curvature of a Circle,* Tangential and Normal Components of Acceleration, The Binormal, Radius of Curvature

9.4: Partial Derivatives-Functions of Two Variables, Level Curves, Level Surfaces, Higher-Order and Mixed Derivatives, Functions of Three or More Variables, Chain Rule, Generalizations

9.5: Directional Derivative-The Gradient of a Function, A Generalization of Partial Differentiation, Method for Computing the Directional Derivative, Functions of Three Variables, Maximum Value of the Directional Derivative, Gradient Points in Direction of Most Rapid Increase of f

9.6: Tangent planes and Normal Lines-Geometric Interpretation of the Gradient, Tangent Plane, Surfaces Given by $z = f(x, y)$, Normal Line

Module II 24 hrs

9.7: Curl and Divergence-Vector Fields, *definition of curl and divergence*, Physical Interpretations

9.8: Line Integrals-*definition of smooth.closed and simple closed curves*, Line Integrals in the Plane, Method of Evaluation-curve as explicit function and curve given parametrically, Line Integrals in Space, Method of Evaluation, Work, Circulation

9.9: Independence of Path- Conservative Vector Fields, Path Independence, A Fundamental Theorem, *definition of connected,simply connected and multiconnected* *regions,* Integrals Around Closed Paths, Test for a Conservative Field, Conservative Vector Fields in 3-Space, Conservation of Energy

9.10: Double Integral**-** Integrability, Area, Volume, Properties, Regions of Type I and II, Iterated Integrals, Evaluation of Double Integrals (*Fubini theorem*), Reversing the Order of Integration, Laminas with Variable Density—Center of Mass, Moments of Inertia, Radius of Gyration

9.11: Double Integrals in Polar Coordinates**-** Polar Rectangles, Change of Variables: Rectangular to Polar Coordinates,

9.12: Green's Theorem**-** Line Integrals Along Simple Closed Curves, *Green's theorem in plane,* Region with Holes,

9.13: Surface Integral**-** Surface Area, Differential of Surface Area, Surface Integral, Method of Evaluation, Projection of *S* into Other Planes, Mass of a Surface, Orientable Surfaces, Integrals of Vector Fields-*Flux,*

9.14: Stokes's Theorem- Vector Form of Green's Theorem, Green's Theorem in 3-Space-*Stoke's Theorem*, Physical Interpretation of Curl

Module III 21 hrs

9.15:Triple Integral- *definition***,** Evaluation by Iterated Integrals, Applications, Cylindrical Coordinates, Conversion of Cylindrical Coordinates to Rectangular Coordinates, Conversion of Rectangular Coordinates to Cylindrical Coordinates, Triple Integrals in Cylindrical Coordinates, Spherical Coordinates, Conversion of Spherical Coordinates to Rectangular and Cylindrical Coordinates, Conversion of Rectangular Coordinates to Spherical Coordinates, Triple Integrals in Spherical Coordinates

9.16: Divergence Theorem- Another Vector Form of Green's Theorem , *divergence or Gauss' theorem, (proof omitted),* Physical Interpretation of Divergence

9.17: Change of Variable in Multiple Integral- Double Integrals, Triple Integrals

17.1: Complex Numbers**-** definition, arithmetic operations, conjugate, Geometric Interpretation

17.2: Powers and roots-Polar Form, Multiplication and Division, Integer Powers of *z* , DeMoivre's Formula, Roots

17.3: Sets in the Complex Plane- *neighbourhood, open sets, domain, region etc.*

17.4: Functions of a Complex Variable- *complex functions*, Complex Functions as Flows, Limits and Continuity, Derivative, Analytic Functions - *entire functions*

17.5: Cauchy Riemann Equation- A Necessary Condition for Analyticity, *Criteria for analyticity,* Harmonic Functions, Harmonic Conjugate Functions,

17.6:Exponential and Logarithmic function- (Complex)Exponential Function, Properties, Periodicity, *('Circuits' omitted), Complex Logarithm-principal value, properties*, Analyticity

17.7: Trigonometric and Hyperbolic functions- Trigonometric Functions, Hyperbolic Functions, Properties -*Analyticity, periodicity, zeros etc.*

Module IV 14 hrs

18.1: Contour integral- *definition*, Method of Evaluation, Properties, *ML*-inequality. Circulation and Net

18.2: Cauchy-Goursat Theorem- Simply and Multiply Connected Domains, Cauchy's Theorem, *Cauchy–Goursat theorem*, Cauchy–Goursat Theorem for Multiply Connected Domains,

18.3: Independence of Path- *Analyticity and path independence*, *fundamental theorem for contour integral, Existence of Antiderivative*

18.4: Cauchy's Integral Formula- First Formula, Second Formula-*C.I.F. for derivatives*. Liouville's Theorem, Fundamental Theorem of Algebra

References:

1	Soo T Tan: Calculus *Brooks/Cole, Cengage Learning(2010)ISBN 0-534-46579-X*
2	Gilbert Strang: Calculus *Wellesley Cambridge Press(1991)ISBN:0-9614088-2-0*
3	Ron Larson. Bruce Edwards: Calculus(11/e) *Cengage Learning(2018) ISBN: 978-1-337-27534-7*
4	Robert A Adams & Christopher Essex : Calculus *several Variable (7/e) Pearson Education Canada (2010) ISBN:* 978-0-321-54929-7
5	Jerrold Marsden & Anthony Tromba : Vector Calculus (6/e) *W. H. Freeman and Company ISBN 978-1-4292-1508-4*
6	Peter V O'Neil: Advanced Engineering Mathematics(7/e) *Cengage Learning(2012)ISBN: 978-1-111-42741-2*
7	Erwin Kreyszig : Advanced Engineering Mathematics(10/e) John Wiley & Sons(2011) ISBN: 978-0-470-45836-5
8	Glyn James: Advanced Modern Engineering Mathematics(4/e) Pearson Education Limited(2011) ISBN: 978-0-273-71923-6

FOURTH SEMESTER

MTS4 C04:MATHEMATICS-4

5 hours/week	3 Credits	75 Marks[Int.15 + Ext. 60]

Text	Advanced Engineering Mathematics(6/e) : Dennis G Zill *Jones & Bartlett Learning, LLC(2018)ISBN: 978-1-284-10590-2*

Module I	21 hrs

Ordinary Differential Equations

1.1: Definitions and Terminology- definition, Classification by Type, Classification by Order, Classification by Linearity, Solution, Interval of Definition, Solution Curve, Explicit and Implicit Solutions, Families of Solutions, Singular Solution, Systems of Differential Equations

1.2: Initial Value Problems-First- and Second-Order IVPs, *Existence of solution*

1.3: Differential Equations as Mathematical Models- *some specific differential-equation models in biology, physics and chemistry.*

2.1: Solution Curves without Solution-Direction Fields *[‘Autonomous First-Order DEs’ omitted]*

2.2: Separable Equations- definition. Method of solution, losing a solution, An Integral-Defined Function

2.3: Linear Equations-definition, standard form, homogeneous and non homogeneous DE, *variation of parameter technique,* Method of Solution, General Solution, Singular Points, Piecewise-Linear Differential Equation, Error Function

2.4: Exact Equations- Differential of a Function of Two Variables**,** *Criteria for an exact differential,* Method of Solution, Integrating Factors,

2.5: Solutions by Substitution-Homogeneous Equations, Bernoulli’s Equation, Reduction to Separation of Variables

2.6: A Numerical Method- Using the Tangent Line, Euler’s Method *[upto and including Example 2; rest omitted]*